# The Bubble Game: An Experimental Study of Speculation

Sophie Moinas and Sebastien Pouget*†

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*University of Toulouse (Toulouse School of Economics and IAE), Place Anatole France, 31000 Toulouse, France, sophie.moinas@univ-tlse1.fr, spouget@univ-tlse1.fr.

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**Abstract**

We propose a bubble game that involves sequential trading of an asset commonly known to be valueless. Because some traders do not know where they stand in the market sequence, the game allows for a bubble at the Nash equilibrium when there is no cap on the maximum price. We run experiments both with and without a price cap. Structural estimation of behavioral game theory models suggests that quantal responses, uncertainty regarding other traders' rationality, and analogy-based expectations are important drivers of speculation.
**Keywords:** Rational bubbles, irrational bubbles, experiments, cognitive hierarchy model, quantal response equilibrium, analogy-based expectation equilibrium

# 1 Introduction

Historical and recent economic developments such as the South Sea, Mississippi, and dot com price run-up episodes suggest that financial markets are prone to bubbles and crashes. However, to the extent that fundamental values cannot be directly observed in the field, it is very difficult to empirically demonstrate that these episodes actually correspond to mispricings.[1] To overcome this difficulty and study bubble phenomena, economists have relied on the experimental methodology: in the laboratory, fundamental values are induced by the researchers and can thus be compared to asset prices. Starting with Smith, Suchanek and Williams (1988), many researchers document the existence of speculative bubbles in experimental financial markets.[2]

We propose a bubble game that complements Smith et al. (1988) and is simple enough to be analyzed using the tools of (behavioral) game theory. Moreover, it enables to control for the number of trading opportunities thus easing the interpretation of experimental data. The bubble game features a sequential market for an asset that generates no cash flow (and this is announced publicly to all market participants). The price proposed to the first trader in the market sequence is random and the subsequent price path is exogenous and chosen such that most traders do not know where they stand in the market sequence.[3] Traders have limited liability and are financed by outside financiers. At each point in the sequence, an incoming trader has the choice between buying or not at the proposed price. If he declines the offer, the game ends and the current owner is stuck with the asset.

When there is a price cap (consistent with the fact that there is a fi-

[1] In this paper, we define the fundamental value of an asset as the price at which agents would be ready to buy the asset given that they cannot resell it later. See Camerer (1989) and Brunnermeier (2009) for surveys on bubbles.

[2] The design created by Smith, Suchanek and Williams (1988) features a double auction market for an asset that pays random dividends in several successive periods. The subsequent literature refined this design to show that irrational bubbles also tend to arise in call markets (Van Boening, Williams, LaMaster, 1993), with a constant fundamental value (Noussair, Robin, Ruffieux, 2001) and with lottery-like assets (Ackert, Charupat, Deaves, and Kluger, 2006), but tend to disappear when some traders are experienced (Dufwenberg, Lindqvist, and Moore, 2005), when there are futures markets (Porter and Smith, 1995) and when short-sales are allowed (Ackert, Charupat, Church, and Deaves, 2005).

[3] Our set up is inspired by the two-envelope puzzle discussed by Nalebuff (1989) and, especially, Geanakoplos (1992). The Supplementary Appendix available online relates the bubble game to this puzzle as well as to the Saint-Petersburg paradox.

nite amount of wealth in the economy), only irrational bubbles can form: upon receiving the highest potential price, a trader realizes that he is last in the market sequence and, if rational, refuses to buy. Even if not sure to be last in the market sequence, the previous trader, if rational, also refuses to buy because he anticipates that the next trader will know he is last and will refuse to trade. This backward induction argument rules out the existence of bubbles when there is a price cap, if all traders are rational and rationality is common knowledge. By increasing the level of the cap, one increases the number of steps of iterated reasoning needed to rule out the bubble. As a result, varying the level of the cap enables the experimenter to understand how bounded rationality or lack of higher-order knowledge of rationality affect bubble formation. This is of interest in light of the theoretical analyses of Morris, Rob, and Shin (1995) and Morris, Postlewaite, and Shin (1995) who show that lack of common knowledge can have important strategic consequences in particular for bubble formation.

When the price cap is infinite, bubbles can be rational because no trader is ever sure to be last in the market sequence. Proposing an experimental analysis of rational bubbles is difficult because extant theories in which bubbles are common knowledge involve infinite trading opportunities and infinite losses.[4] [5] The bubble game overcomes these difficulties: there is a finite number of trades and the potentially infinite losses are concentrated in the hands of outside financiers who are consequently not part of the experiment.

Our experiment features various treatments depending on the existence and the level of a price cap. Subjects participate in only one treatment and in a one-shot game.[6] Our experimental results are as follows. First, bubbles

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[4]Such an infinite number of trading opportunities may derive from infinite horizon models (see, for example, Tirole (1985) for deterministic bubbles, Blanchard (1979) and Weil (1987) for stochastic bubbles, Abreu and Brunnermeier (2003), and Doblas-Madrid (2010) for clock games), or from continuous trading models (see Allen and Gorton, 1993).

[5]The theoretical analyses of Allen, Morris, and Postlewaite (1993), and Conlon (2004) show that rational bubbles can occur with a finite number of trading opportunities and without exposing participants to potentially infinite losses. These analyses however involve asymmetric information regarding the asset cash flows. In order to be in line with the literature on experimental bubbles, we design an experiment in which there is no asymmetric information on the asset payoff. Because trading is not continuous, asset prices as well as potential gains and losses have to grow without bounds for a bubble to be sustained at equilibrium (see Tirole, 1982).

[6]The Supplementary Appendix reports two robustness experiments. In the first experiment, the same treatments are used but the game is now repeated five times with

arise whether or not there is a cap on prices. Bubbles thus form even if they would be ruled out by backward induction. Second, the propensity for a subject to enter a bubble increases with the distance between the offered price and the maximum price. We refer to this phenomenon as a snow-ball effect, and show that it is related to a higher probability not to be last and to a higher number of steps of iterated reasoning.

To better understand speculative behavior, we estimate various specifications of two models of bounded rationality, departing from the Nash equilibrium in different ways: the Subjective Quantal Response Equilibrium (hereafter SQRE) of Rogers, Camerer and Palfrey (2009), and the Analogy-Based Expectation Equilibrium (hereafter ABEE) of Jehiel (2005). Both models are able to account for the snowball effect that is observed in our data for all treatments. Estimating SQRE and its various nested models, we show that speculation in the bubble game is related to quantal responses rather than to Cognitive Hierarchies (hereafter CH). The best-fitting specification in this class is a Heterogeneous Quantal Response Equilibrium (hereafter HQRE) that generalizes the QRE of Mc Kelvey and Palfrey (1995) to take into account the fact that subjects ignore the precise level of others' rationality.

The ABEE offers an interesting complementary point of view on the bubble game. Bianchi and Jehiel (2011) suggest that the ABEE logic can generate bubbles in an environment in which they do not arise if all traders are commonly known to be perfectly rational. According to the ABEE logic, agents bundle nodes at which others move into analogy classes. Agents then form correct expectations for the average behavior within each class. [7] The ABEE concept is relevant here because various types of analogy classes arise quite naturally in the bubble game. We can thus estimate what type of analogy classes best fits our data and is thus most likely to be important for bubble formation. Our estimations show that an ABEE with two analogy classes, one including the traders who know they are not last and the other including the remaining traders, has a fit that is not significantly different from that of the HQRE. This indicates that both heterogeneous noisy best

stranger matching. In the second experiment, a one-shot game experiment is organized with executive MBA students. Experience with the game or in business slightly reduces but does not eliminate the propensity to enter into bubbles.

[7]When estimating ABEE, we follow Huck, Jehiel, and Rutter (2010) and consider that agents play noisy best responses to their beliefs regarding other traders' behavior. See Jehiel and Koessler (2008) for an extension of the ABEE to the incomplete information case.

responses and analogy classes are important drivers of speculation in the bubble game.

The rest of the paper is organized as follows. The next section compares the bubble game to the previous literature. Section 3 presents the bubble game and the Bayesian Nash predictions. Section 4 derives the behavioral game theory predictions using SQRE, and ABEE logics. The empirical results are in Section 5. Section 6 concludes and provides potential extensions.

# 2 Literature review

The bubble game in which agents trade sequentially can be viewed as a generalization of the centipede game in which not all players know where they stand in the sequence.[8] [9] The bubble game shares some features with the centipede game. On the one hand, because of limited liability, the sum of traders' potential gains increases as the bubble grows. On the other hand, when there is a price cap, the game can be solved by backward induction.

There are however several important differences between the bubble game and the centipede game. First, our generalization enables the existence of a bubble equilibrium when there is no cap on prices, without relying on an infinite horizon game. Second, since traders play only once, there is no reputation building considerations in the bubble game. Third, in the bubble game, traders are offered a price at which they can buy. This price reveals information which enables them to perform inferences regarding their position. This informational ingredient is not present in the centipede game.

These conceptual differences have important consequences from an experimental point of view. First, one can perform a bubble experiment in an environment in which there actually exists a bubble equilibrium. Second, the absence of reputational issues eliminates one potential explanation for behavior that is not really relevant for bubbles from an empirical perspective. Third, the informational aspect of our game opens the scope for behavioral

[8]See, for example, Mc Kelvey and Palfrey (1992) for an experimental analysis of the centipede game.

[9]This is related to the absent-minded centipede game proposed by Dulleck and Oechssler (1997): in a classic centipede game, some agents suffer from imperfect recall and may not realize that they have reached the end of the game. The bubble game is different in the sense that even traders with perfect recall may not know where they stand in the sequence, and that price information received by traders enables further inference regarding their position.

regularities that are not present in the centipede game. In particular, we show that QRE is better at explaining speculative behavior than CH which is the opposite to what has been previously found on the centipede game.[10] The relative importance of quantal responses compared to cognitive hierarchies for bubble formation is a novel empirical finding that opens interesting perspectives for the understanding of speculative behavior.

Our experimental analysis is also related to Lei, Noussair and Plott (2001) and to Brunnermeier and Morgan (2010). [11] Lei, Noussair and Plott (2001) use Smith et al. (1988)'s design and show that, even when they cannot resell and realize capital gains, some participants still buy the asset at a price which exceeds the sum of the expected dividends. This behavior is consistent with risk-loving preferences or violation of dominance. We extend Lei et al. (2001)'s analysis in the sense that, in our design, i) risk preferences cannot explain speculation by agents who are offered the maximum price, and ii) one can observe the behavior of traders who need to perform one, two and even more steps of iterated reasoning to find out that speculating is not an equilibrium.

Brunnermeier and Morgan (2010) study clock games both from a theoretical and an experimental standpoint. These clock games can indeed be viewed as metaphors of "bubble fighting" by speculators, gradually and privately informed of the fact that an asset is overvalued. Speculators do not know if others are already aware of the bubble. They have to decide when to sell the asset knowing that such a move is profitable only if enough speculators have also decided to sell. Their experimental investigation and ours share two common features. First, the potential payoffs are exogenously fixed, that is, there is a predetermined price path. Second, there is a lack of common knowledge over a fundamental variable of the environment. In Brunnermeier and Morgan (2010), the existence of a bubble is not common knowledge. In our setting, the existence of the bubble is common knowledge but traders' position in the market sequence is not. There are several differences between our approach and theirs. A first difference is the time di-

[10]See, for example, the experimental investigations of the centipede game by Mc Kelvey and Palfrey (1992) who apply the QRE logic, and by Kawagoe and Takizawa (2010) who apply the CH logic and argue that it better fits the data than the QRE logic.

[11]A related analysis of bubbles is offered by Palfrey and Wang (2011) who experimentally study speculation due to traders' differential interpretation of public signals. A recent working paper by Asparouhova, Bossaerts, and Tran (2011) study bubbles in a laboratory experiment in which the asset payoff is determined by the result of a centipede game.

mension. The theoretical results tested by Brunnermeier and Morgan (2006) depend on the existence of an infinite time horizon. They implement this feature in the laboratory by randomly determining the end of the session. By contrast, we design an economic setting in which there could be bubbles in finite time with finite trading opportunities, even if traders act rationally. A second difference is that our experimental design also enables the study of irrational bubbles. A third difference is that we rationalize the formation of rational and irrational bubbles by showing that bounded rationality models can explain observed behavior.

# 3 The bubble game

This section proposes a simple experimental design in which bubbles may or may not be ruled out by backward induction. This design features a sequential market for an asset whose fundamental value is commonly known to be 0. There are three traders in the market.[12] Trading proceeds sequentially. Each trader is assigned a position in the market sequence and can be first, second or third with the same probability $\frac{1}{3}$. Traders are not told their position in the market sequence but can infer some information when observing the price at which they are offered to buy.

Prices are exogenously given and are powers of 10.[13] For simplicity, we do not include the issuer of the asset in the present experimental design. The first trader is offered to buy at a price $P_1 = 10^n$. The power $n$ follows a geometric distribution of parameter $\frac{1}{2}$, that is $\mathbb{P}(n = j) = \frac{1}{2}^{j+1}$, with $j \in \mathbb{N}$. The geometric distribution is useful from an experimental point of view because it is simple to explain and implies that the conditional probability to be last in the market sequence is equal to 0 if the proposed price is 1 or 10, and is equal to $\frac{4}{7}$ otherwise.[14] If a trader decides to buy the asset at price

[12] We could have designed an experiment with only two traders per market. However, this would have required higher payments for bubbles to be rational. Indeed, the conditional probability to be last would be higher. We could also have chosen to include more than three traders per market. We decided not to do so in order to have a sufficiently high number of observations at the different price levels.

[13] We have chosen prices to be powers of 10 in order for the profit in case of a successful speculation to compensate for the loss incurred in case of a failed speculation. If there were more traders in the market sequence, the probability to be last would decrease and price explosiveness could be lower.

[14] The probabilities to be first, second or third conditional on the prices, which are

$P_t$, he proposes the asset to the next trader at a price $P_{t+1} = 10P_t$.

In order to prevent participants from discovering their position in the market sequence by hearing other subjects making choices or by measuring the time elapsed since the beginning of the game, subjects play simultaneously. Once $P_1$ has been randomly determined, the first, second and third traders are simultaneously offered prices of $P_1$, $P_2$, and $P_3$, respectively.[15] If they decide to buy, they automatically try and resell the asset.

Each trader is endowed with 1 unit of capital. Additional capital may be required in order to buy the asset at price $P_t > 1$. This additional capital (that is, $P_t - 1$) is provided by an outside financier. The experimenter plays the role of the outside financier for all players. Payoffs are divided between the trader and the financier in proportion of the capital initially invested: a fraction $\frac{1}{P_t}$ for the trader, and a fraction $\frac{P_t-1}{P_t}$ for the financier. Consider a trader who decides to buy the asset at price $P_t$. When he is able to resell, his final wealth is 0 which corresponds to the fundamental value of the asset. The outside financier also ends up with 0. When the trader is able to resell the asset, he gets $\frac{1}{P_t}$ percent of the proceed $P_{t+1} = 10P_t$ and thus ends up with a final wealth of 10. The outside financier ends up with $10P_t - 10$.[16]

The separation of payoffs between traders and outside financiers allows implementing limited liability in the experiment: the maximum potential loss of a trader is 1. The potentially infinite gains and losses are incurred by financiers. However, financing all the traders and also playing the role of the issuer, the experimenter faces a maximum total payment, per cohort of 3 subjects, of 20. This maximum payment occurs when all subjects decide to enter the bubble. The experimenter is thus not subject to bankruptcy risk.

The timing of the bubble game is depicted in Figure 1.[17] Speculating is

computed using Bayes' rule, are given to the participants in the Instructions.

[15] This experimental procedure corresponds to the strategy method. When a trader does not accept to buy the asset, subsequent traders end up with their initial wealth whatever their decision. The advantage of this method is that we can observe traders' speculation decision even if a bubble does not actually develop.

[16] When traders are self-financed, payoffs' absolute values are scaled up by $P_t$. Introducing traders' limited liability and outside financiers undoes this scaling. This change has some relevance from a practical point of view because most traders do have limited liability and invest other people's money. This change can also have some consequences for behavior in our experiment (as well as in practice): the stakes being smaller than when they are self-financed, traders might have more incentive to enter into bubbles.

[17] This timing does not correspond to the extensive form of the game. Indeed, it leaves aside the issue of which player is first, second, or third. The extensive-form game is

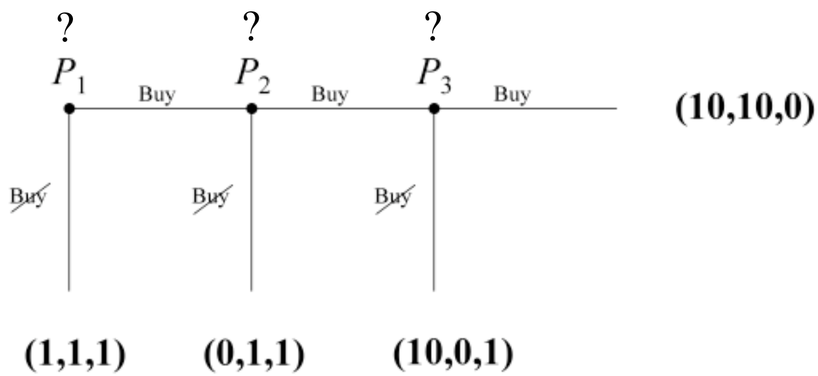



Figure 1: Timing of the bubble game.

The bubble game features a sequential market in which traders are protected by limited liability and financed by outside financiers. Question marks emphasize the fact that traders are equally likely to be first, second or third, and to be offered to buy at prices P1, P2, or P3, respectively. This figure displays traders' payoff only. In case of successful speculation, traders' payoff is 10 because prices are powers of 10 and traders invest one unit of capital. Appendix A provides the extensive-form game for the two-trader case.

profitable for trader $j$ if the following individual rationality (IRj) condition is satisfied:

$$[1 - \mathbb{P}(\text{last})] \times \mathbb{P}(\text{next trader buys}) \times U_j(10) + \mathbb{P}(\text{last})\, U_j(0) \geq U_j(1),$$

where $U_j(.)$ is the trader's utility function.

In order to study how traders' rationality influences bubble formation, we introduce a cap $K$ on the first price (that translates into a cap of $100K$ on the highest potential price in the bubble game). The Bayesian Nash equilibrium is as follows. If $K$ is finite, upon being proposed a price of $100K$, an agent understands that he is last in the market sequence. Consequently, his individual rationality condition cannot be satisfied and he refuses to buy. Anticipating this refusal, agents who are proposed lower prices also refuse to buy, even if they are not sure to be last in the market sequence. At the Bayesian Nash equilibrium, a bubble never forms, being ruled out by backward induction. This establishes a connexion between the bubble game and the previous experiments initiated by Smith et al. (1988) that focus on irrational bubbles.

The bubble game complements this previous literature because we can vary $K$ to study how the number of iterated steps of reasoning needed to

provided in Appendix A for the two-trader case. When there is no cap on the first price, it includes an infinite number of nodes.

reach the Nash equilibrium influences speculation. When the proposed price is $P = 100K$, an agent knows that he is last and there is no iterated step of reasoning needed. When the proposed price is $P = 10K$, a subject knows that he is not first in the market sequence (he can be second or third). At equilibrium, he has to anticipate that the next trader in the market sequence (if any) would not accept to buy the asset. One step of iterated reasoning is thus needed to derive the equilibrium strategy. More generally, when the proposed price is $1 \leq P \leq 100K$, the required number of iterated steps of reasoning is $log_{10}\left(\frac{100K}{P}\right)$. In order to study whether this required number of iterated steps of reasoning affects bubble formation, we have chosen to experimentally study treatments in which $K$ equals 1, 100, and 10,000.

Another interesting aspect of our design is that we can let $K$ go to infinity. A bubble can arise at equilibrium if the (IRj) condition is satisfied for all traders on the market. It is straightforward to show that, if traders anticipate that other traders speculate, there indeed exists increasing and concave utility functions $U_j(.)$ for which this (IRi) condition holds. The bubble game thus offers an economic environment in which rational bubbles can form despite the number of trades being finite and the existence of the bubble being common knowledge. Hence our paper contributes to the literature on rational bubbles by showing that neither infinite trading horizon (see Blanchard (1979) and Tirole (1982)) nor infinite trading speed (see Allen and Gorton (1993) and Abreu and Brunnermeier (2003)) are necessary for common knowledge rational bubbles to exist. The possibility of equilibrium bubbles in our setting arises because no trader is ever sure to be last in the market sequence.[18]

In order to show that the bubble equilibrium is meaningful, we now check whether financiers are willing to fuel the bubble. It is clear that, if the same financier provides capital to all traders (as it is the case in the experiment), his total expected profit would be negative. However, we show below that, if each trader has a different outside financier, these financiers may have an interest in providing capital to traders. Assuming that a financier has an initial wealth denoted by $W$, his individual rationality condition (IRf) is written as:

$$[1 - \mathbb{P}(\text{last})]\, \mathbb{P}(\text{next trader buys})\, U_f(W + 10P_t - 10) + \mathbb{P}(\text{last})\, U_f(W - P_t + 1) \geq U_f(W),$$

[18] It is straightforward to show that a no-bubble equilibrium always exists. The Supplementary Appendix proves the existence of a bubble equilibrium for the risk neutral and constant relative risk aversion cases. It also offers a more extensive theoretical analysis of the bubble game including a welfare analysis.

for all $P_t$. $U_f(.)$ is the financier's utility function. It is again straightforward to show that, if financiers expect that all traders speculate, there exist functions $U_f(.)$ for which the (IRf) condition holds.

The experimental protocol is as follows. Our baseline experiment includes a total of 234 subjects. Subjects are junior and senior undergraduates in Business Administration at the University of Toulouse. Each subject participates in only one session and receives a 5-euro show-up fee. Each session includes only one replication of the trading game. Subjects' risk aversion is measured thanks to a procedure inspired from Holt and Laury (2002). We adjust their questionnaire in order to match the set of possible decisions to the decisions subjects actually face in our experiment.[19] The minimum, median, maximum, and average gains in the experiment are respectively 0, 1, 10, and 3.35 euros (not including the show-up fee). The instructions for the case where $K = 10,000$ are in Appendix B.

Our experimental protocol is summarized in Table I.

| Session | # Replications | # Subjects | cap on initial price, $K$ | Bayesian Nash Equilibrium |
|---|---|---|---|---|
| 1, 5, and 9 | 1 | 60 | 1 | no-bubble |
| 2, 6, and 10 | 1 | 54 | 100 | no-bubble |
| 3, 7, and 11 | 1 | 63 | 10,000 | no-bubble |
| 4, 8, and 12 | 1 | 57 | $+\infty$ | no-bubble or bubble |

Table 1: Experimental protocol.

# 4 Behavioral game theory predictions

This section analyzes the game with various behavioral game theory models. The next section structurally estimates these models using data from the

[19] The questionnaire is composed by a table with 14 decisions. For each decision $i$, subjects may choose between the riskless option A, which is to receive 1 euro for sure, or the risky option B, which is to receive 10 euros with probability $\frac{i}{14}$, or 0 euro with probability $\frac{14-i}{14}$. This questionnaire features what Harrison, List and Towe (2007) refer to as a higher frame: a risk-neutral agent switches to the risky option B in the upper part of the table. It gives us a precise estimation of the willingness to accept the bets at stake in the bubble game.

bubble game. Two types of models appear relevant in our context: the Subjective Quantal Response Equilibrium (hereafter SQRE) of Rogers, Camerer and Palfrey (2009), and the Analogy-Based Expectation Equilibrium (hereafter ABEE) of Jehiel (2005). On the one hand, the SQRE is of interest here since i) it is based on the concept of noisy best-response that proved useful to explain failures of backward induction in previous experiments (see Camerer (2003)), and ii) it allows for heterogeneity in agents' rationality that could be an important driver of bubble formation. On the other hand, the ABEE is relevant here because i) speculation heavily relies on beliefs' formation that is at the heart of this equilibrium concept, and ii) candidates for analogy classes arise naturally in the bubble game.

## 4.1 Subjective Quantal Response Equilibrium

According to the SQRE logic, an agent's payoff responsiveness, denoted by $\lambda_{i,s}$, depends on his type $i$ and on his level of sophistication, denoted by $s$. SQRE then involves a stochastic choice model whereby the agent's propensity to choose an action has a logistic form that depends on the expected profit of this action given his information, and on his payoff responsiveness. The expected payoff from buying the asset conditional on being proposed a price $P$ is denoted by $u_{i,s}(B|P)$. The expected payoff from not buying the asset is denoted by $u_{\varnothing}$. The probability that agent $i$ buys the asset after being proposed a price $P$ is thus: $\Pr_{i,s}(B|P) = \frac{e^{\lambda_{i,s} u_{i,s}(B|P)}}{e^{\lambda_{i,s} u_{i,s}(B|P)} + e^{\lambda_{i,s} u_{\varnothing}}}$.

In the SQRE logic, agent $i$'s payoff responsiveness is given by: $\lambda_{i,s} = \lambda_i + \gamma s$, where it is commonly known that $\lambda_i$ is uniformly distributed over the interval $[\Lambda - \frac{\epsilon}{2}, \Lambda + \frac{\epsilon}{2}]$, and where $\gamma$ represents the sensitivity of an agent's payoff responsiveness to his level of sophistication. The level of sophistication $\tilde{s}$ follows a Poisson distribution with density function $f$. Let $\tau$ denote the average level of sophistication.

Finally, in the SQRE logic, agents may not have the same understanding of the overall population of players. In particular, it is assumed that an agent with sophistication $s$ cannot imagine that other agents can have a sophistication greater than $s - \theta$. The agent's truncated beliefs about the fraction of h-level players is thus $g_s(h) = \frac{f(h)}{\sum_{i=0}^{s-\theta} f(i)}$.

Overall, SQRE has five parameters: $\Lambda$, the basic payoff responsiveness; $\epsilon$, the uncertainty surrounding the basic payoff responsiveness; $\tau$, the average level of sophistication; $\gamma$, the sensitivity of payoff responsiveness to sophistication; and $\theta$, the imagination parameter.

One advantage of using the SQRE is that it nests various interesting behavioral game theory models. In particular, when $\Lambda = 0$, $\epsilon = 0$, $\gamma = +\infty$ and $\theta = 1$, SQRE boils down to the Cognitive Hierarchy model (hereafter CH) developed by Camerer, Ho and Chong (2004) with only one free parameter $\tau$. The CH model states that agents best-respond to mutually inconsistent beliefs: they believe that all other agents are at most one level of sophistication below them. Moreover, agents with a level of sophistication $s = 0$ choose each available action with an equal probability. Alternatively, when $\epsilon = 0$, $\theta = +\infty$, and when $\gamma = 0$ or $\tau = 0$, SQRE corresponds to the Quantal Response Equilibrium (hereafter QRE) of Mc Kelvey and Palfrey (1995) with only one free parameter $\Lambda$. The QRE takes into account the fact that players make mistakes but it retains beliefs' consistency. At equilibrium, players are responsive to payoffs to the extent that more profitable actions are chosen more often. Using the SQRE enables us to study whether noisy best-responses with beliefs consistency or best-responses with beliefs inconsistencies best explain speculation in the bubble game.

Another advantage of the SQRE is that the QRE and CH model can be extended to take into account heterogeneity across agents. The CH can be extended to a (discretized) Truncated Quantal Response Equilibrium (hereafter TQRE) by freeing the parameter $\gamma$. As a result, we have a CH model in which agents, instead of best-responding, have a payoff responsiveness that increases with their level of sophistication. Likewise, the QRE can be extended to a Heterogeneous Quantal Response Equilibrium (hereafter HQRE) by freeing the parameter $\epsilon$. We then have a QRE in which agents do not know for sure what the exact level of payoff responsiveness of another agent is.[20] Both of these extensions can prove useful to understand whether heterogeneity across agents plays a role for bubble formation.

A last advantage of the SQRE is that is can be used to estimate whether overconfidence matters for bubble formation. For some values of the limited imagination parameter $\theta$, agents underestimate the average level of sophistication in the population of players, a bias referred to as the better-than average effect in the psychology literature. Starting from the CH model and freeing the parameter $\theta$, we can estimate whether agents suffer from limited imagination. We call this model the overconfidence CH (hereafter OCH).

[20] Uncertainty about other traders' payoff responsiveness can be interpreted as uncertainty about their level of risk aversion.

We now apply the SQRE to the bubble game. For brevity, we focus here on the QRE and CH models for treatments with a finite price cap $K$.[21]

We first study the QRE. After being proposed a price $P = 100K$, a trader perfectly infers that he is last in the market sequence and buys with probability $\Pr(B|P = 100K) = \frac{1}{1+e^{\lambda}}$. After observing a price $P = 10K$, a trader infers that he has a specific probability, denoted by $q(K, P = 10K)$, not to be last. He correctly anticipates that the probability to buy of the last trader is not equal to zero. His expected payoffs from buying is $u(B|P = 10K) = q(K, P = 10K) \times \frac{1}{1+e^{\lambda}} \times 10$. His probability to buy is therefore $\Pr(B|P = 10K) = \frac{1}{1+e^{\lambda\times\left(1-\frac{10q(K,P=10K)}{1+e^{\lambda}}\right)}}$, which is greater than $\Pr(B|P = 100K)$. Applying this logic backward, we find the predicted probability that a trader buys at all potential prices. This analysis shows that the QRE predicts a snowball effect: traders are more likely to enter the bubble when they are further away from the maximum price.

Consider now the CH model. When proposed a price of $100K$, a trader knows he is last in the sequence. Consequently, only level-0 traders buy, with probability $\frac{1}{2}$. Given that there is a fraction $f(0) = \frac{\tau^0 e^{-\tau}}{0!}$ of such traders in the population, the probability to observe a trader buying at this price is: $\Pr(B|P = 100K) = \frac{1}{2}e^{-\tau}$.

When a trader is being proposed a price $P = 10K$, he infers that he is penultimate in the sequence. If he is a level-0 trader, he buys with probability $\Pr_{\tilde{s}=0}(B|P = 10) = \frac{1}{2}$. If he is a level-$s$ player with $s \geq 1$, he thinks that the next trader observing the price $100K$ is a level-$h$ with probability $g_s(h) = \frac{f(h)}{\sum_{i=0}^{s-1} f(i)}$. Consequently, his expected profit if he buys is: $u_{\tilde{s}=s\geq 1}(B|P = 10K) = q(K, P = 10K) \times \frac{f(0)}{\sum_{i=0}^{s-1} f(i)} \times \frac{1}{2} \times 10$. The trader is strictly better off buying if and only if $\sum_{i=0}^{s-1} \frac{\tau^i}{i!} < 5q(K, P = 10K)$. Given that $\sum_{i=0}^{s-1} \frac{\tau^i}{i!}$ is strictly increasing in $s$, there exists a (potentially infinite) threshold $s^* \geq 1$ such that only traders with a level below or equal to $s^*$ buy. This is because higher level traders have a more accurate perception of the distribution of lower-level t ypes. Finally, given the actual distribution

[21] The computations for the general specification of SQRE as well as for the case in which $K = +\infty$ are available in the Supplementary Appendix. Extending CH and QRE to incorporate traders' heterogeneity as modeled in TQRE and HQRE does not change the underlying logic. For the case in which $K = +\infty$, we derive predictions by relying on equilibrium conjectures that are realized at equilibrium as it is the case for the derivation of the Nash equilibrium.

of traders' types, the probability to observe a trader buying at a price of $P = 10K$ is: $\Pr\left(B|P = 10K\right) = \frac{1}{2}e^{-\tau} + \sum_{s=1}^{s=s^*}\left(\frac{\tau^s}{s!}e^{-\tau}\right)$. As before, the rest of the model is solved backward. The CH model predicts that a snowball effect may arise because $s^*$ increases with the distance from the maximum price.

## 4.2 Analogy-Based Expectation Equilibrium

According to the ABEE logic, agents use simplified representations of their environment in order to form expectations. In particular, agents are assumed to bundle nodes at which other agents make choices into analogy classes. Agents then form correct beliefs concerning the average behavior within each analogy class. Following Huck, Jehiel, and Rutter (2010), we consider that agents apply noisy best-responses to their beliefs. Our version of the ABEE can thus be viewed as a generalization of the QRE in which agents do not hold consistent beliefs.

In the bubble game, two types of analogy classes arise naturally. On the one hand, traders may use only one analogy class, assuming that other traders' behavior is the same across all potential prices. On the other hand, traders may use two analogy classes: one class (Class I) that includes prices at which traders are sure not to be last in the market sequence, the other (Class II) that includes the remaining prices (at which traders think they may be last or know they are last).

We now apply the ABEE to the bubble game. For brevity, we restrict our attention to the case in which the price cap is $K = 1$ (the other cases are addressed in the Supplementary Appendix). Let $p_1$, $p_2$, and $p_3$ denote the actual probability that a trader buys after observing prices equal to 1, 10, and 100, respectively. Let $\Pr(B|P = 1)$, $\Pr(B|P = 10)$, and $\Pr(B|P = 100)$ be the corresponding probabilities as (mis)perceived by traders using analogy classes.

We start by analyzing the one-class ABEE. A trader after observing a price $P = 100$ knows he is last. Consequently, his probability to buy is $p_3 = \frac{1}{1+e^{\lambda}}$. A trader after observing a price $P = 10$ has the following expected payoff from buying: $u(B|P = 10) = 10 \times \Pr(B|P = 100)$. Because of the use of one analogy class, we have $\Pr(B|P = 100) = \frac{p_1+p_2+p_3}{3}$.[22] The

[22]The probability $\frac{1}{3}$ corresponds to the ex-ante probability to observe prices of 1, 10 or 100

probability to buy after $P = 10$ is therefore: $p_2 = \frac{1}{1+e^{\lambda\left(1-\frac{10}{3}(p_1+p_2+p_3)\right)}}$. A trader after observing a price $P = 1$ has an expected payoff of $u(B|P = 1) = 10 \times \Pr(B|P = 10) = 10\frac{p_1+p_2+p_3}{3}$.The probability to buy is therefore: $p_1 = p_2$. This analysis leaves us with a system of equations that can be solved numerically to find $p_1$, $p_2$, and $p_3$.

We now turn to the two-class ABEE. As before, a trader after observing a price $P = 100$ buys with a probability $p_3 = \frac{1}{1+e^{\lambda}}$. A trader after observing a price $P = 10$ has the following expected payoff from buying: $u(B|P = 10) = 10 \times \Pr(B|P = 100)$. Because Class II is a singleton, we have $\Pr(B|P = 100) = p_3$. Thus, we have $p_2 = \frac{1}{1+e^{\lambda(1-10p_3)}}$. A trader after observing a price $P = 1$ uses Class I to form his expectations and thus has an expected payoff of $u(B|P = 1) = 10 \times \Pr(B|P = 10) = 10\frac{p_1+p_2}{2}$. The probability to buy is therefore: $p_1 = \frac{1}{1+e^{\lambda(1-5(p_1+p_2))}}$. The system of equations can again be solved numerically. Overall, one can show that ABEE can display an additional snowball effect due to the bundling of nodes into analogy classes, and can thus explain speculation in the bubble game even when there is a price cap.[23]

# 5 Empirical Results

## 5.1 The determinants of speculation in the bubble game

To gain insights on bubble formation, we study individual decisions to buy the overvalued asset. Figure 2 plots, for each treatment, the proportion of buy decisions for each price level. The number of times a given price has been proposed is indicated at the bottom of the bar. Below the horizontal axis, we explicitly indicate, for each price, the number of steps of reasoning required to reach equilibrium as well as the conditional probability not to be last. Let's first focus on the treatments with a cap on the first price. The rather high probabilities to buy in these treatments (Figure 2, Panels A, B and C) are inconsistent with Nash equilibrium. Keeping the probability not to be last constant, it seems that traders are more likely to buy when more steps of reasoning are required (for example, compare the proportion of buy decisions when traders are sure not to be last in Figure 2, Panels A, B, and C). This is line with previous experimental results on the centipede

[23]We would also find a snowball effect if traders were best responding to their beliefs, that is, if $\lambda = +\infty$.

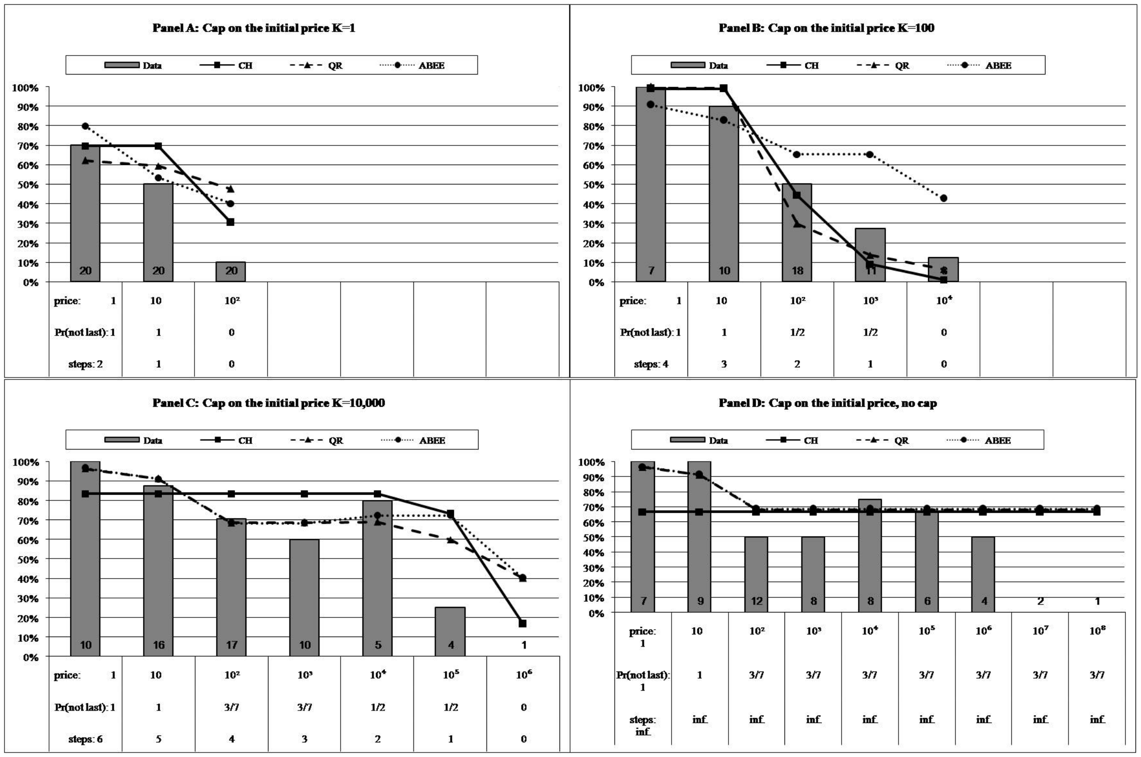



Figure 2: Probability of a Buy decision, depending on the initial price, the probability not to be last and the number of steps of iterated reasoning. Predictions from behavioral game theory models use the parameters of interest estimated on the entire data set.

game. Also, keeping the number of steps of reasoning constant, it seems that traders are more likely to buy when their probability not to be last increases (for example, compare the proportion of buy decisions for 3 and 4 steps of reasoning in Figure 2, Panel B and C). This is a new empirical result that could not meaningfully be obtained in the centipede game because the probability not to be last is equal to 1 for each node except the last one at which it is 0. This result indicates that there is some elements of rationality in subjects' decisions.

We now turn to the treatment in which there is no cap on the first price (Figure 2, Panel D). First, subjects who are sure not to be last always buy the asset which indicates a higher propensity to speculate than when there is a price cap (for example, compare the proportion of buy decisions when traders are sure not to be last in Figure 2, Panels D, and C). This reveals another facet of subjects' rationality. This result appears interesting in light of the fact that, when the probability not to be last is lower than 1, subjects are not more likely to speculate when there is no cap than when there is one despite the same difference in the required number of reasoning steps than in the case in which the probability is equal to one. A Wilcoxon rank sum test indicates that the proportion of buy decision when subjects are offered a price of 1 or 10 is significantly higher when there is no cap (100%) than when there is one (77%) (the p-value is 0.034). This result however does not hold if we compare the cases $K = +\infty$ and $K = 10,000$ (92%) (the p-value is 0.261). Besides, there is no difference in the probability to buy when a subject has a probability not to be last equal to $\frac{3}{7}$ or $\frac{1}{2}$ between the cases $K = +\infty$ (54%) and $K = 10,000$ (64%) (the p-value is 0.367).

Second, when there is no cap and when prices are 100 or above, if participants coordinate on the same equilibrium, their decisions should be the same for all price levels. In line with this hypothesis, using a Wilcoxon rank sum test, we cannot reject the fact that the probability to buy is the same after observing prices of 100, 1,000, and 10,000 (57%), and after observing higher prices (46%) with a p-value of 0.517 (this test keeps the required number of steps constant and equal to infinity). Again, these results on the treatment with no cap cannot be obtained in an experiment with the centipede game, and underline the interest of our design for the study of speculation.

In order to fine-tune our statistical analysis, we run a logit regression.[24] The propensity to buy the overvalued asset is explained by several variables. Variables 1 through 7 are indicators. Variables 1 and 2 indicate that the subject has 1 or 2 required steps of reasoning to reach equilibrium and a probability to be last strictly included between 0 and 1, and equal to 0, respectively. Variable 3 and 4 indicate that the subject has 3 or more required steps of reasoning and a probability to be last strictly included between 0 and 1, and equal to 0, respectively. Variable 5 interacts Variable 1 with the indicator that the cap is 10,000. Variable 6 interacts Variable 3 with the indicator that there is no cap. Variable 7 interacts Variable 3 with the indicator that the cap is 10,000. The last two variables are the individual degree of risk aversion and the offered price.[25] The constant reflects the propensity to speculate of subjects who are proposed to buy at the maximum price ($100K$). This is useful because, since these subjects are expected not to buy, their probability to buy can be viewed as the incompressible level of noise in our data.[26]

[24]The results are the same if we run a probit regression.

[25]The coefficient of risk aversion is computed assuming a constant relative risk aversion utility function as in Holt and Laury (2002).

[26]The indicators that the required number of steps is 1 or 2, that it is 3 or more, that the probability not to be last is strictly included between 0 and 1, and that is 0 are not included in our regressors because these variables are collinear with the interaction variables that

The results are in Table II. We first focus on the subjects who know they are last in the market sequence. We can reject the hypothesis that these subjects never enter the bubble. Out of the 29 subjects who knew they were last, three bought the asset. This number is low but it is not zero. This result is in line with the findings of Lei, Noussair and Plott (2001) that subjects were buying an overvalued asset even when prohibited to resell. These agents can be viewed in our framework as "step 0" subjects. In page 853, Lei et al. (2001) report that 6 out of 36 subjects made at least one dominated transaction.[27] This proportion (16.7%) is slightly higher than our proportion of dominated choices (10.3%), maybe reflecting the more complicated framework used in their experiment. We now complement their results by studying the behavior of subjects who are further away from the maximum price and who have more chances not to be last.

The regression analysis provides other interesting results. First, the coefficients of Variables 1 through 4 are significantly positive at the one percent level. This indicates that subjects who know they are last buy significantly less than the others. Second, a Wald test indicates that the difference between the coefficients of Variables 1 and 3 is statistically significant with a p-value of 0.07. Moreover, the difference between the coefficients of Variables 2 and 4 is statistically significant with a p-value smaller than 0.01. Keeping constant the probability to be last, subjects are thus more likely to buy when there are 3 or more steps than when there are 1 or 2 steps of reasoning. Third, the difference between the coefficients of Variables 1 and 2 is marginally significant with a p-value of 0.11, and the difference between

we include. For example, by definition of step 0, the sum of variables 1 and 2 is equal to the indicator that the required number of steps is 1 or 2. Also, given that all the subjects who were sure not to be last decided to buy in the no cap treatment, we do not introduce in our regressors the indicator that a subject has an infinite number of required steps of reasoning. Finally, Variables 5, 6, and 7 enable us to test whether the level of the cap has an effect on the propensity to speculate over and above the number of required steps of reasoning and the probability to be last. We cannot include more interaction variables with the level of the cap because these interaction variables would have no variability. For example, interacting Variable 1 with the no cap indicator would generate a variable that always equals 0.

[27]We focus here on the experiment of Lei, Noussair and Plott (2001) during which subjects could participate in several markets, the so-called TwoMarket/NoSpec treatment. This is because, in this treatment, subjects could participate actively in the experiment without being forced to participate in the bubble. This provides a lower bound for the number of subjects who make mistakes in the market.

| | Coefficient | p-value |
|---|---|---|
| Constant | -1.84 | 0.005 |
| 1 : $I_{Step=1\ or\ 2}$ x $I_{0<P(last)<1}$ | 1.80 | 0.013 |
| 2: $I_{Step=1\ or\ 2}$ x $I_{P(last)=0}$ | 2.60 | 0.000 |
| 3: $I_{Step>=3}$ x $I_{0<P(last)<1}$ | 2.82 | 0.000 |
| 4: $I_{Step>=3}$ x $I_{P(last)=0}$ | 5.65 | 0.000 |
| 5 :$I_{Step=1\ or\ 2}$ x $I_{0<P(last)<1}$ x $I_{cap=10,000}$ | 0.57 | 0.464 |
| 6: $I_{Step>=3}$ x $I_{0<P(last)<1}$ x $I_{no\ cap}$ | -0.32 | 0.556 |
| 7: $I_{Step>=3}$ x $I_{P(last)=0}$ x $I_{cap=10,000}$ | -0.96 | 0.443 |
| Degree of risk aversion for consistent choices | - 0.62 | 0.249 |
| Price | -0.00 | 0.451 |
| Log likelihood | -116.22 | |
| Number of observations | 234 | |

Table 2: Logit Regression of the Purchase Decision.
$I_x$ is an indicator variable that takes the value 1 if condition $x$ is true. The variable Step represents the number of steps of reasoning needed to rule out bubbles. The variable P(last) is the probability to be last in the market sequence.

the coefficients of Variables 3 and 4 is statistically significant with a p-value of 0.01. Keeping constant the required number of reasoning steps, subjects are more likely to buy when they are less likely to be last. Finally, we find no additional effect on the cap, the coefficients of Variables 5 through 7 being insignificant: univariate differences exhibited in Figure 2 are thus well captured by our Variables 1 to 4.

These results uncover a snowball effect: the propensity to enter bubbles increases with the required number of reasoning steps and with the probability not to be last. This snowball effect is not present at the Bayesian Nash equilibrium (whether there is a cap or not) but is displayed by the behavioral game theory models discussed in the previous section. The next subsection proposes a structural estimation of these models in order to better understand the nature of speculation in the bubble game.

## 5.2 Estimating behavioral game theory models of speculation

Our results so far suggest that some players have bounded rationality and that the formation of bubbles is related to a snowball effect. To account for

| | | Session | | | | |
|---|---|---|---|---|---|---|
| | | All | No cap | Cap K=10,000 | Cap K=100 | Cap K=1 |
| Data | | | | | | |
| | Sample size | 234 | 57 | 63 | 54 | 60 |
| | Av. probability buy | 60% | 67% | 75% | 54% | 43% |
| Nash Equilibrium | | | | | | |
| | Av. probability buy | 24% | 100% | 0% | 0% | 0% |
| | Log L | -1 114.46 | -175.00 | -432.89 | -267.10 | -239.47 |
| CH | | | | | | |
| | Tau | 0.5 | 0.4 | 1.1 | 3.9 | 0.5 |
| | Av. probability buy | 65% | 66% | 82% | 48% | 57% |
| | Log L | -143.61 | -36.28 | -34.36 | -30.02 | -36.65 |
| | 95% CI | [0.3 - 0.7] | [0.1 - 0.8] | [0.4 - 1.9] | [1.2 - 4.0] | [0.2 - 4.1] |
| TQRE | | | | | | |
| | Tau | 3.7 | 4.4 | 5.3 | 3.8 | 0.5 |
| | Gamma | 0.1 | 0.1 | 0.1 | 4.8 | 34.3 |
| | Av. probability buy | 68% | 72% | 78% | 53% | 57% |
| | Log L | -137.45 | -31.86 | -30.64 | -29.77 | -36.65 |
| | 95% CI Tau | [2.7 - 4.7] | [3.2 - 6.0] | [1.5 - 7.5] | [2.2 - 12.1] | [0.2 - 4.3] |
| | 95% CI Gamma | [0.1 - 0.1] | [0.1 - 0.1] | [0.1 - 50.0] | [0.1 - 33.1] | [4.1 - 37.2] |
| QRE | | | | | | |
| | Lambda | 0.3 | 0.4 | 0.4 | 2.7 | 0.1 |
| | Av. probability buy | 69% | 75% | 78% | 45% | 56% |
| | Log L | -135.12 | -31.25 | -30.17 | -29.84 | -39.79 |
| | 95% CI | [0.2 - 0.4] | [0.3 - 7.8] | [0.3 - 0.8] | [0.3 - 3.0] | [0.0 - 3.2] |
| HQRE | | | | | | |
| | Lambda | 1.1 | 0.4 | 0.4 | 2.3 | 1.8 |
| | Epsilon | 0.4 | 0 | 0 | 1.4 | 0.4 |
| | Av. probability buy | 64% | 75% | 78% | 53% | 46% |
| | Log L | -128.91 | -31.25 | -30.17 | -25.88 | -33.21 |
| | 95% CI Lambda | [0.4 - 1.9] | [0.3 - 0.5] | [0.3 - 3.1] | [0.5 - 3.1] | [1.1 - 2.9] |
| | 95% CI Epsilon | [0.0 - 0.6] | [0.0 - 0.0] | [0.0 - 2.8] | [0.0 - 4.8] | [0.4 - 1.0] |
| ABEE - 1 class | | | | | | |
| | Lambda | 0.2 | 0.3 | 0.3 | 0.2 | 0.1 |
| | Av. probability buy | 64% | 72% | 75% | 63% | 57% |
| | Log L | -141.42 | -31.77 | -31.67 | -32.92 | -40.03 |
| | 95% CI | [0.1 - 0.3] | [0.2 - 0.4] | [0.2 - 0.4] | [0.2 - 0.4] | [0.0 - 0.2] |
| ABEE - 2 classes | | | | | | |
| | Lambda | 0.3 | 0.4 | 0.4 | 0.3 | 0.4 |
| | Av. probability buy | 66% | 75% | 79% | 68% | 58% |
| | Log L | -132.50 | -31.32 | -30.97 | -31.98 | -37.73 |
| | 95% CI | [0.3 - 0.4] | [0.3 - 0.4] | [0.2 - 0.5] | [0.2 - 0.5] | [0.2 - 0.7] |

Table 3: Estimations and goodness of fit of behavioral game theory models.

these phenomena, we estimate models that explicitly incorporate bounded rationality: the Subjective Quantal Response Equilibrium (hereafter SQRE) of Rogers, Camerer and Palfrey (2009), and the Analogy-Based Expectation Equilibrium (hereafter ABEE) of Jehiel (2005). For each model, we estimate the parameters of interest using maximum likelihood methods for the entire data set as well as for each treatment separately. Confidence intervals are computed using a bootstrapping procedure: using the empirical distribution of the observed data, we resample 10,000 data sets on which the parameters of interest are re-estimated. We then choose the 2.5 and 97.5 percentile points values to construct 95% confidence intervals. When comparing the fits of two models, we use a likelihood ratio test when the models are nested, and Vuong (1989)'s test when they are not. [28]

Table III reports our estimation results, and Figure 3 displays the sta-

[28] Under the null hypothesis, the probability distribution of the log-likelihood ratio statistic used to test nested models is approximated by a Chi-squared distribution with degrees of freedom equal to the difference between the numbers of parameters in the two models, while the probability distribution of the Vuong's statistic used to test non-nested models is a standard normal dis tribution.

tistical tests for various models' comparisons. The first two lines of Table III describe our data, namely, the number of observations, and the observed average probability to buy. The next two lines show the predictions and log-likelihoods of the Nash equilibrium under risk neutrality.[29] The mean choices are generally far away from the Nash equilibrium; the observed probability to buy is too low when there exists a bubble-equilibrium, and too high when it does not exist.

Table III then provides the predictions and log-likelihoods of the various SQRE and ABEE models including the Cognitive Hierarchy (hereafter CH) and its extension, the Truncated Quantal Response Equilibrium (hereafter TQRE), and the Quantal Response equilibrium (hereafter QRE) and its extensions, the Heterogeneous QRE (hereafter HQRE) and the one- and two-classes ABEE. For brevity, the estimations of the overconfidence CH model (hereafter OCH) and of the general SQRE model are given in the Supplementary Appendix.

The results are as follows. The average level of sophistication $\tau$ of the CH, estimated on the entire data set, is 0.5. This is in line with the median estimates reported by Camerer, Ho and Chong (2004) that lie between 0.7 and 1.9, but is a little low. This low estimated $\tau$ suggests a high proportion of level-0 players, around 60%. Interestingly, what drives this result is not really the fact that traders enter too much into bubbles when they should not. Indeed the fact that there is only around 10% of subjects who buy when they know they are last in the market sequence suggests a proportion of level-0 players equal to 20%. What explains the high estimated proportion of level-0 players is rather the fact that subjects do not buy as much as expected by the cognitive hierarchy model (with a higher average sophistication level) when there is no cap on the initial price or when the cap is large. This can be seen on Figure 2 that plots the predictions of the CH model using the best-fitting value of $\tau$ estimated on the entire data set. CH embeds the better than average effect to the extent that traders believe that no other agent does as many steps of reasoning as them. Our estimations of OCH (reported in the Supplementary Appendix) show that we cannot reject the hypothesis that $\theta = 1$, its CH restriction. Overconfidence is thus an important feature that enables the CH logic to fit our data pretty well.

[29]We consider that traders coordinate on the bubble equilibrium when there is no cap on the initial price. The no-bubble Nash equilibrium has a lower log-likelihood. In order to compute the likelihoods, we assume that players choose non-equilibrium strategies with a probability of 0.0001.

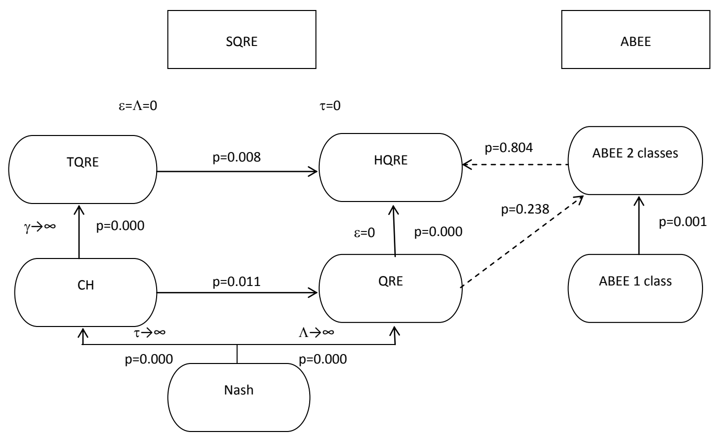



Figure 3: Statistical tests for various models' comparisons using parameters estimated across all treatments. The arrows point towards the model with the highest likelihood. Plain and dotted lines indicate, respectively, significant and insignificant differences in likelihood.

The payoff responsiveness $\Lambda$, estimated on the entire data set, is 0.3. This is consistent with the results of Mc Kelvey and Palfrey (1995) who report estimates between 0.15 and 3.3. As was also the case for the CH model, such a low value of the responsiveness parameter is required not so much to explain why subjects buy when they should not (that is, when the offered prices are close to the price cap, if any) but to explain why they do not buy that much when they should (that is, when the offered prices are low).

As shown in Figure 3 that displays the statistical tests for various models' comparisons, the QRE fits our data better than the CH model (p-value=0.01 for the overall data). This result is interesting because Camerer, Ho and Chong (2004) show that CH fits better than QRE for a wide variety of games. This suggests that there is a specific aspect to the nature of speculation in the bubble game. Moreover, this result is also at odd with those of Kawagoe and Takizawa (2010), who compare the goodness of fit of both models in laboratory experiments of the centipede game. In order to understand why QRE fits better than CH in the bubble game, it is interesting to focus on the treatment with $K = 1$. Indeed, this treatment corresponds to a specific centipede game with three agents playing once. In this treatment, CH appears to fit better than QRE (p-value=0.046). The other treatments with $K > 1$ are not centipede games because some agents do not know what their position is. In this case, the information revealed by prices enable traders to better infer their chances not to be last and affect their expected payoffs. For these treatments, QRE fits better than CH most of the time (p-values are 0.491, 0.064, and 0.000 for $K$ equals 100, 10,000, and $+\infty$, respectively). Given that the informativeness of prices is a relevant feature from an empirical point of view, this result demonstrates again the interest of our design in better understanding the nature of speculation.

Looking at Figure 2, it seems that QRE better captures the drop in the probability to buy for prices $P \geq 100$. In the QRE, since costlier mistakes are less likely, this model is able to capture the drop in players' expected utility from buying: when they are proposed a price $P \geq 100$, the conditional probability to be third is greater than or equal to $\frac{4}{7}$, whereas, when they are proposed a price of 1 or 10, the conditional probability to be third is zero. This informational feature is present in our design but not in the centipede game, and has behavioral consequences in the bubble game.

We then estimate generalizations of the CH and QRE models. As Figure 3 indicates, TQRE and HQRE improve on CH and QRE models, respectively. This suggests that taking into account heterogeneity in payoff responsiveness enables to better fit data from the bubble game. However, HQRE still fits better than TQRE, asserting the fact that cognitive hierarchies are not a crucial ingredient to understand speculation in the bubble game. [30] We thus conclude that the nature of speculation in the bubble game is related to less than perfect payoff responsiveness as well as uncertainty concerning this responsiveness.

We finally estimate the ABEE models that assume traders form expectations within analogy classes. We find that the two-class ABEE fits the bubble game data better than the one-class ABEE. This confirms that informational aspects related to the inference on the probability not to be last play an important role in bubble formation in our game. Moreover, the fit of the two-class ABEE is not significantly different from the one of the QRE and of the HQRE, suggesting that both (heterogeneous) limited payoff responsiveness and analogy-classes are important ingredients in understanding bubble formation.

Finally, for most of the behavioral game theory models, the parameters of interest, estimated for each treatment, display some variability: point

[30] As shown in the Supplementary Appendix, the goodness of fit of HQRE is not statistically different than the one of the general SQRE

estimates in Table III often vary by an order of magnitude. This indicates that most of the theoretical models fail to capture all the strategic aspects of the bubble game. On the contrary, the two-class ABEE estimates appear quite stable across treatments. This suggests that analogy classes play an important role in understanding speculation as hypothesized by Bianchi and Jehiel (2011).

# 6 Conclusion

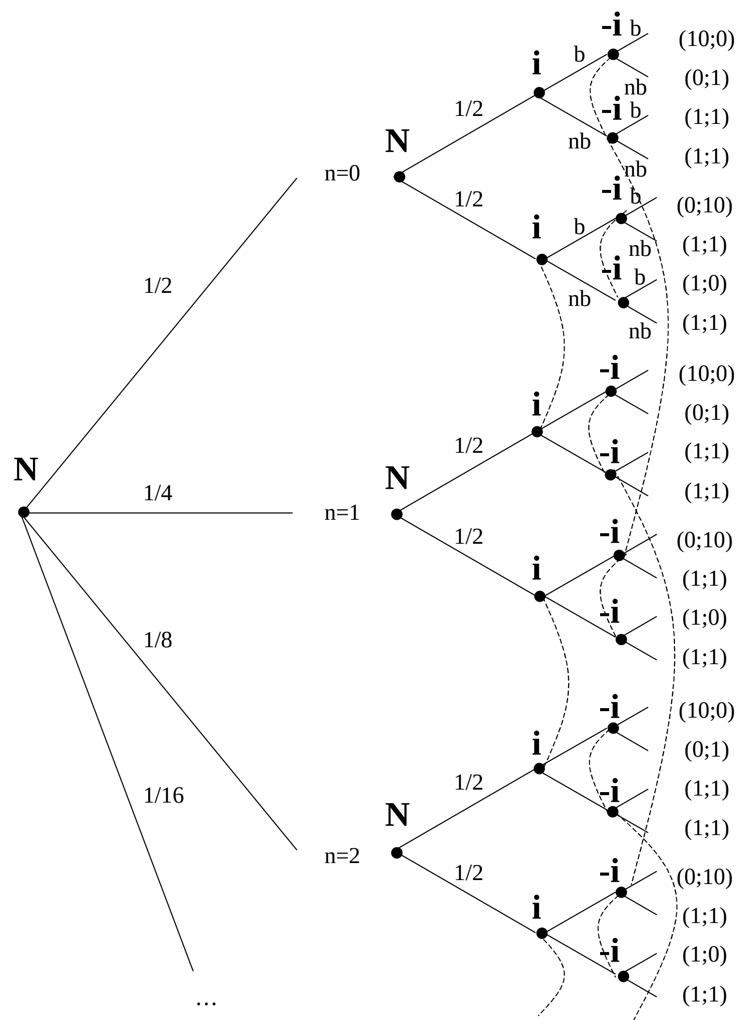
This paper proposes a novel experimental design to study speculative behavior in laboratory experiments: a bubble game in which agents trade sequentially and do not always know where they stand in the sequence. Our game has a no bubble Nash equilibrium when there is a finite price cap, and an additional bubble equilibrium when there is no price cap. To better understand the nature of speculation, we estimate various behavioral game theory models.

Analyzing our experimental data, we show that speculation increases with the number of steps of iterated reasoning needed to reach equilibrium as well as with the probability that a subject is not last in the market sequence as revealed by the offered price. Maximum likelihood estimations suggest that the nature of speculation in the bubble game is related to heterogeneous quantal responses (Rogers, Palfrey, and Camerer, 2009) and to analogy-based expectations (Jehiel, 2005).

The experimental setting proposed in the present paper opens several avenues of research. It would be interesting to study whether the occurrence of bubbles (rational and irrational) vary with the number of traders, the introduction of risk in the underlying asset payoff, and the level of transparency (one could proxy for transparency by setting a non-null probability that a trade is publicly announced). It would also be interesting to extend the experimental setting to cases in which the price path and the timing are left at the discretion of traders. This would allow testing whether traders are able to coordinate on a price path and a timing that sustains rational bubbles.

# 7 Appendix

Appendix A: Extensive form of the game with two players




At each node, Nature (N), player $i$ or player $-i$ choose an action. $(x; y)$ represents the payoffs; $x$ for player $i$, and $y$ for player $-i$. Dotted lines relate nodes that are observationally equivalent. $b$ refers to the buy decision, $nb$ to the refusal decision.

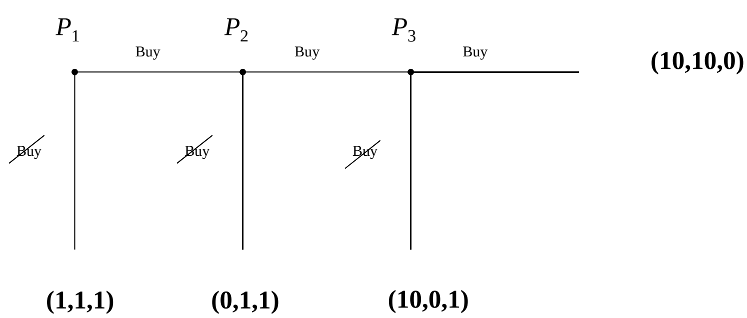
## Appendix B: Instructions for the case where $K = 10,000$

Welcome to this market game. Please read carefully the following instructions. They are identical for all participants. Please do not communicate with the other participants, stay quiet, and turn off your mobile phone during the game. If you have questions, please raise your hand. An instructor will come and answer.

As an appreciation for your presence today, you receive a participation fee of 5 euros. In addition to this amount, you can earn money during the game. The game will last approximately half an hour, including the reading of the instructions.

**Exchange process**

To play this game, we form groups of three players. Each player is endowed with one euro which can be used to buy an asset. Your task during the game is thus to choose whether you want to buy or not the asset. This asset does not generate any dividend. If the asset price exceeds one euro, you can still buy the asset. We indeed consider that a financial partner (who is not part of the game) provides you with the additional capital and shares profits with you according to the respective capital invested. The market proceeds sequentially. The first player is proposed to buy at a price $P_1$. If he buys, he proposes to sell the asset to the second player at a price which is ten times higher, $P_2 = 10 \times P_1$. If the second player accepts to buy, the first player ends up the game with 10 euros. The second player then proposes to sell the asset to the third trader at a price $P_3 = 10 \times P_2 = 100 \times P_1$. If the third player buys the asset, the second player ends up the game with 10 euros. The third player does not find anybody to whom he can sell the asset. Since this asset does not generate any dividend, he ends up the game with 0 euro. This game is summarized in the following figure



At the beginning of the game, players do not know their position in the market sequence. Positions are randomly determined with one chance out of three for each player to be first, second or third.

**Proposed prices**

The price $P_1$ that is proposed to the first player is random. This price is a power of 10 and is determined as follows:

| Price $P_1$ | Probability that this price is realized |
|---|---|
| 1 | 1/2 (50%) |
| 10 | 1/4 (25%) |
| 100 | 1/8 (12.5%) |
| 1,000 | 1/16 (6.3%) |
| 10,000 | 1/16 (6.3%) |

Players decisions are made simultaneously and privately. For example, if the first price $P_1 = 1$ has been drawn, the prices that are simultaneously proposed to the three players are: $P_1 = 1$ for the first player, $P_2 = 10$ for the second player, and $P_3 = 100$ for the third player. Identically, if the first price $P_1 = 10,000$ has been drawn, the prices that are simultaneously proposed to the three players are: $P_1 = 10,000$ for the first player, $P_2 = 100,000$ for the second player, and $P_3 = 1,000,000$ for the third player.

The prices that you are been proposed can give you the following information regarding your position in the market sequence:

- if you are proposed to buy at a price of 1, you are sure to be first;

- if you are proposed to buy at a price of 10, you have one chance out of three to be first and two chances out of three to be second in the market sequence;

- if you are proposed to buy at a price of 100 or 1,000, you have one chance out of seven to be first, two chances out of seven to be second, and four chances out seven to be last in the market sequence;

- if you are proposed to buy at a price of 10,000, you have one chance out of four to be first, one chance out of four to be second, and two chances out four to be last.

- if you are proposed to buy at a price of 100,000, you have one chance out of two to be second, and one chance out of two to be third.

- if you are proposed to buy at a price of 1,000,000, you are sure to be last.

In order to preserve anonymity, a number will be assigned to each player. Once decision will be made, we will tell you (anonymously) the group to which you belong, your position in the market sequence, if you are proposed to buy, and your final gain.

Do you have any question?

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