

Knowledge Cycles and Corporate Investment

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May 2019

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 - Low investment *despite* high valuations – Q (Gutierrez and Philippon (2017) and Crouzet and Eberly (2018))
 - Weak relation between investment and Q (Peters and Taylor (2017))
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- Open (relevant) questions:
 - How does knowledge interact with physical investment and valuation?
 - Does knowledge affect the investment- Q relation? How?

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 - Can depreciate very fast (obsolescence)
 - Effect on profits is highly uncertain
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- Our view: Knowledge results from **exploration and experimentation**

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- Knowledge = Information about current technology’s quality
 - Passive learning-by-doing (i.e., passage of time)
 - Active **Experimentation** – learning-by-investing
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- Key: Knowledge is a by-product of economic activities (Arrow (1962))
- **Explore** new tech when firms are confident that current tech is
 - Poor quality (unprofitable)
 - High quality (more entry eroding profits) – innovation needs

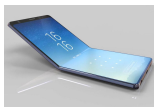
Illustration: Samsung recent knowledge cycle



- **Explore and adopt “color” screen**
- Investment/experimentation (knowledge ↑)
- Confidence that quality is high ↑
- Competitive pressure ⇒ innovation needed



- **Explore and adopt “touch” screen**
- Investment/experimentation (knowledge ↑)
- Confidence that quality is high ↑
- Competitive pressure ⇒ innovation needed



- **Explore and adopt “foldable” touch screen**
- Investment/experimentation (knowledge ↑)
- Confidence that quality is low ↑
- Explore something else (or fix)

- Such knowledge cycles are present in most firms (in various forms)

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- Knowledge creates a **distortion** between investment and Q
 - Over- or Under-investment in some states
 - Investment- Q relation concentrates in low or intermediate knowledge (i.e., early cycle)
 - Varies with competitive pressure and uncertainty about tech quality (noise)
- Rationalize some recent empirical puzzles
- Interesting (new) economic forces in investment models

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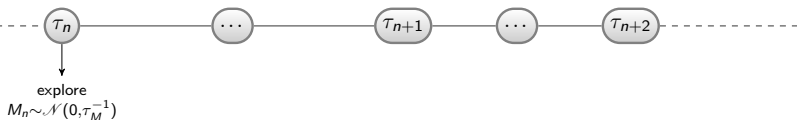
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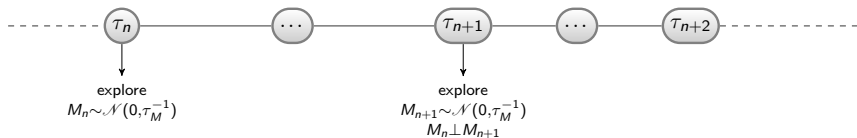
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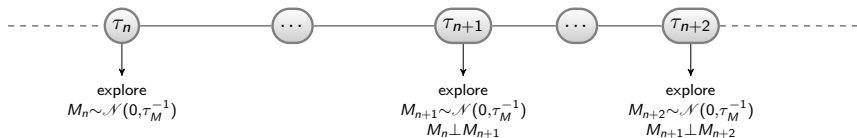
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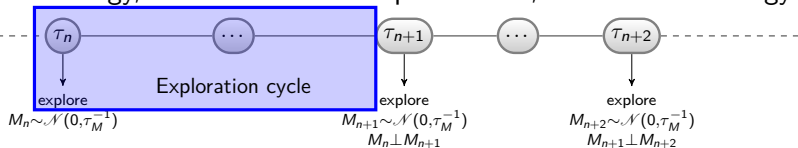
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Investment as Knowledge Acquisition

- The firm can actively learn by “experimenting through investment”. Investment ($i_t \equiv I_t/K_t > 0$) creates an informative signal flow:

$$dS_t = \mathbf{1}_{i_t > 0} \left(\frac{\tau_S^{1/2} i_t^{1/2}}{\Omega_{t-}^{1/2}} M + \epsilon_t \right), \quad \epsilon_t \sim \mathcal{N}(0, 1), \quad \text{i.i.d.}$$

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- Based on passive $(A_t)_{t \geq 0}$ and active signals $(S_t)_{t \geq 0}$ the firm obtains an estimate $\widehat{M}_t = \mathbb{E}_t[M]$ of quality, with error variance, $\Omega_t = \mathbb{V}_t[M]$

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- The t -stat ratio, $Z \equiv \widehat{M}/\Omega^{1/2}$, is a sufficient statistic for the firm's stock of knowledge about its current technology:

$$dZ_t = \underbrace{\tau_A/2 Z_t dt + \tau_A^{1/2} d\widehat{B}_t}_{\text{passive learning}} + \underbrace{Z_{t-} \left((1 + \tau_S i_t)^{1/2} - 1 \right) + (\tau_S i_t)^{1/2} \widehat{\epsilon}_t}_{\text{experimenting}} - \underbrace{Z_{t-} \mathbf{1}_{t=\tau}}_{\text{knowledge reset}}$$

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- To introduce competitive pressures we assume that new firms enter the industry as knowledge about the current technology improves:

$$dN_t/N_t = \phi Z_t^2 dt$$

Firm Value

- Firm is risk-neutral and discounts its cash flow at rate r ; its value is:

$$\max_{\{\tau_n\}, i_{t+s} \geq 0} \mathbb{E}_t \left[\int_0^\infty e^{-rs} \Pi_{t+s} ds - \sum_{k \geq 0} \gamma_{t+\theta_k} e^{-r\theta_k} \right]$$

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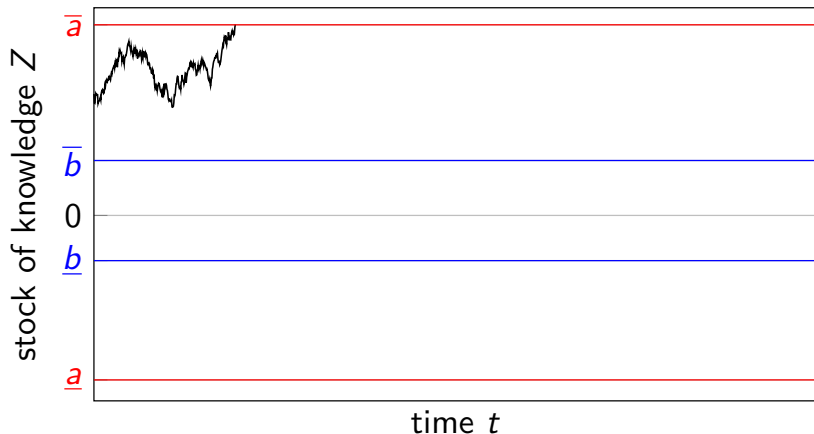
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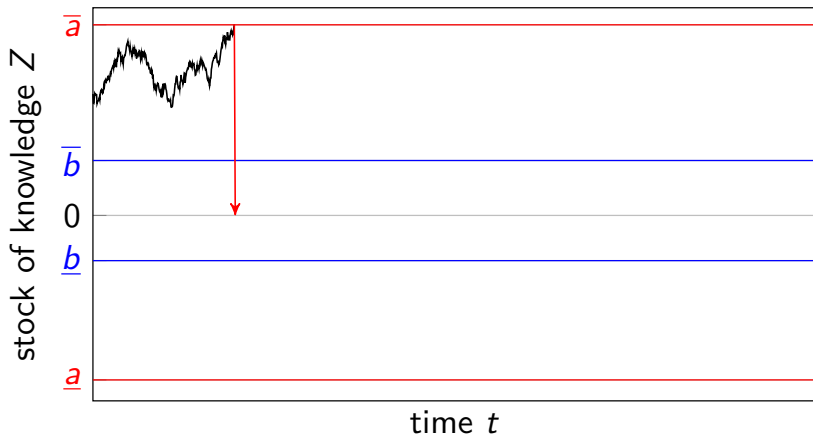
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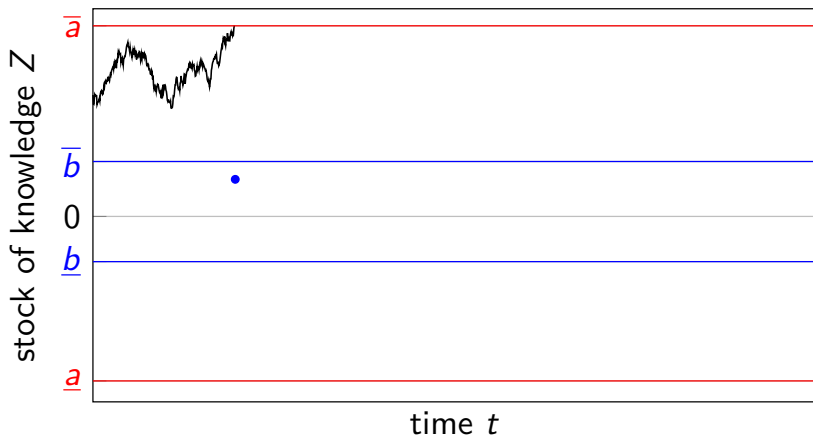
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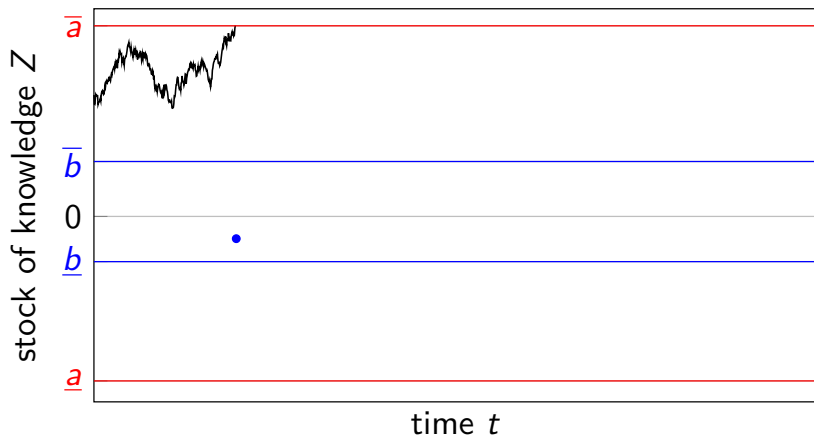
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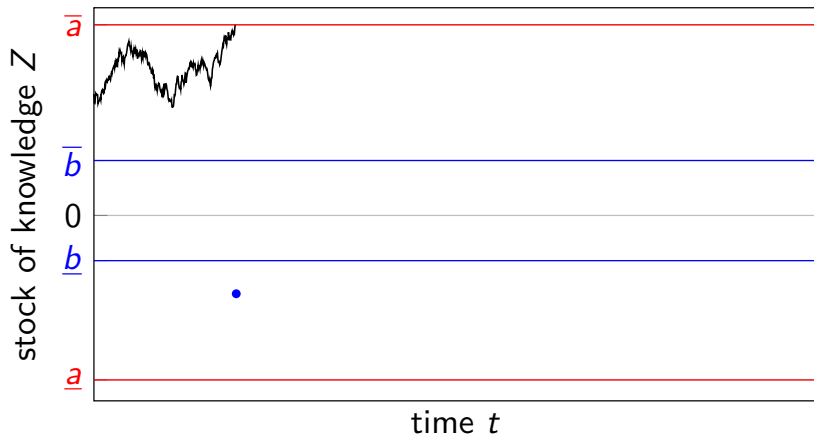
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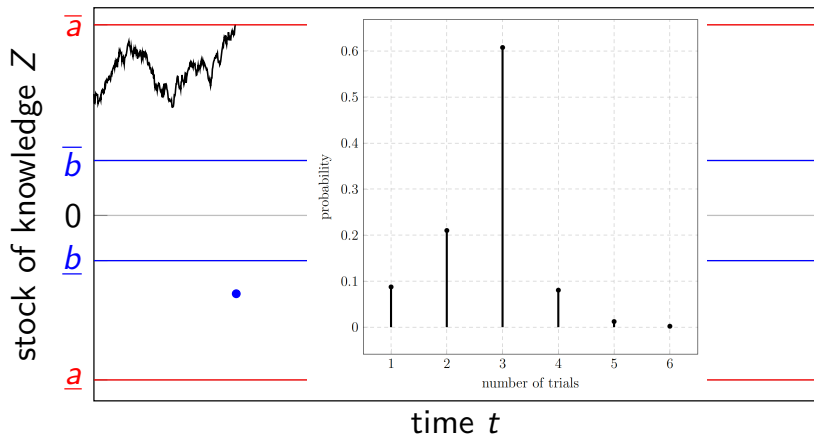
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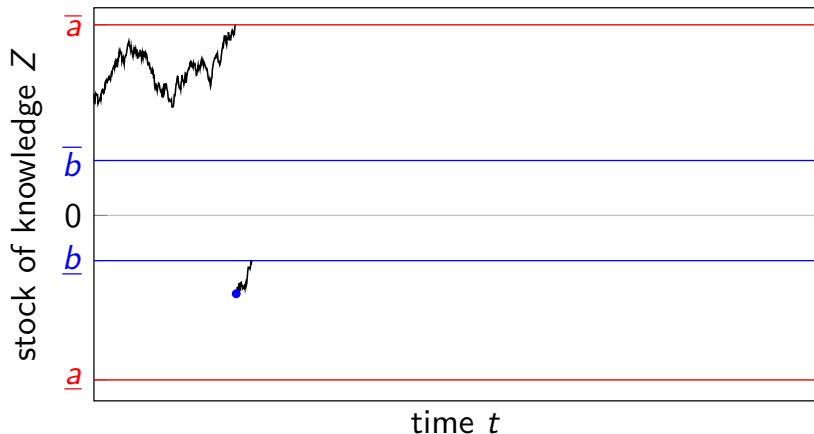
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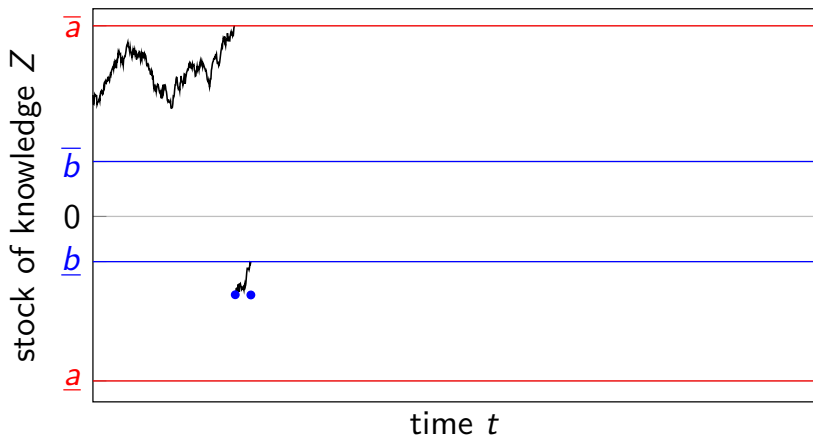
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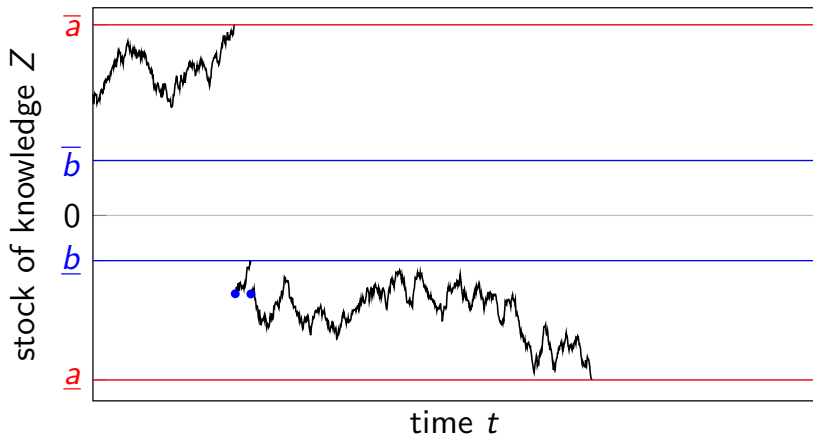
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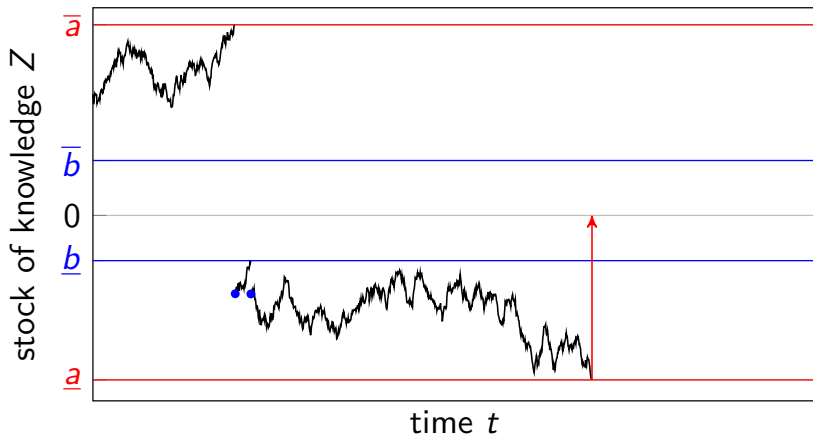
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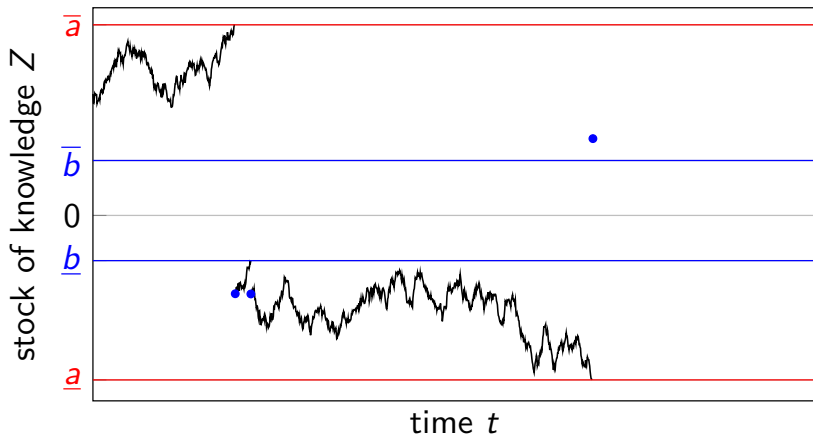
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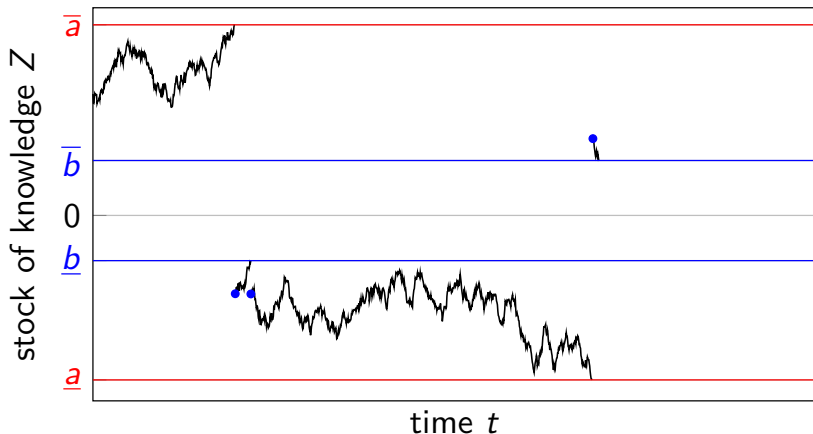
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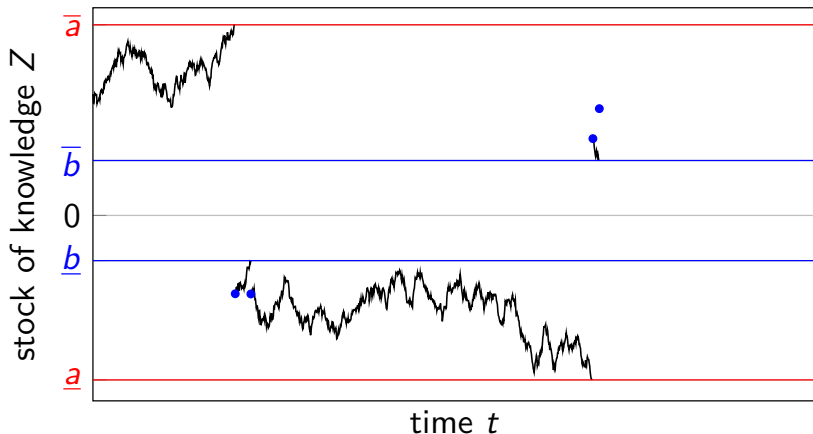
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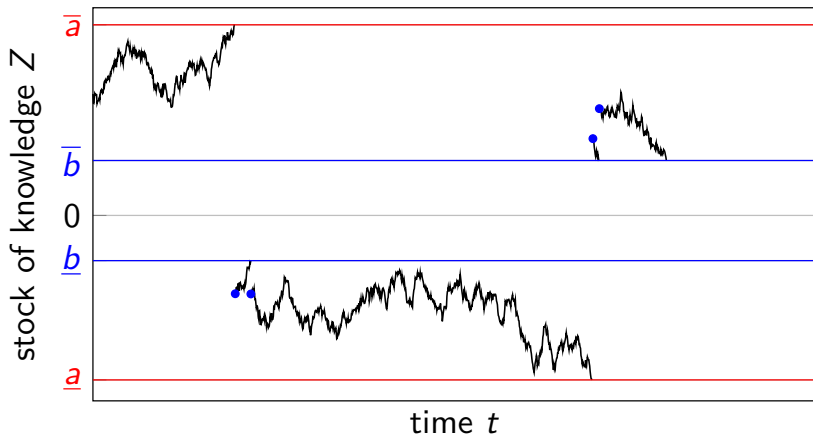
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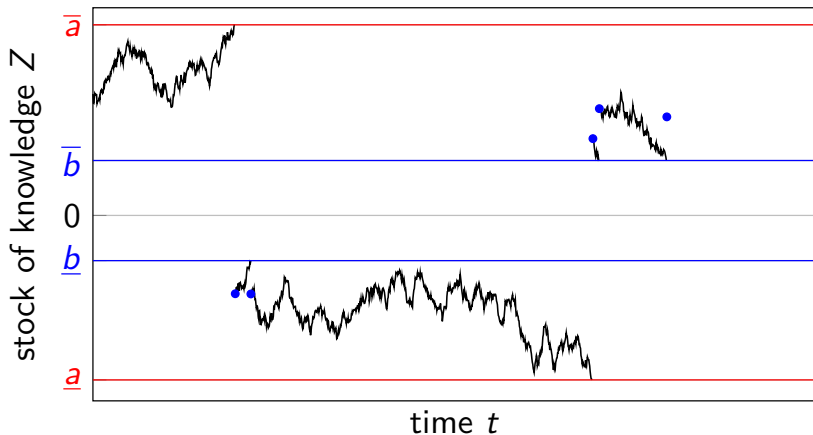
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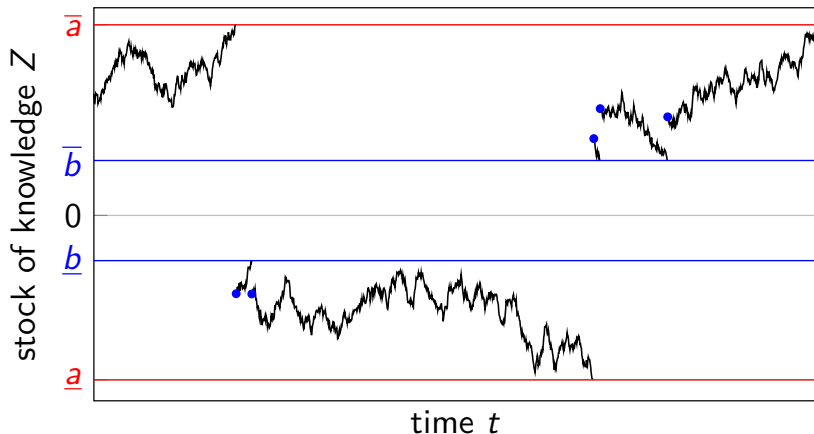
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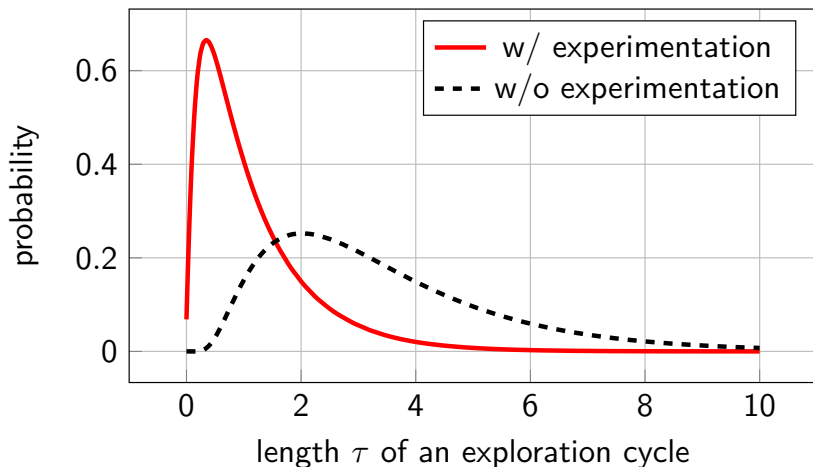
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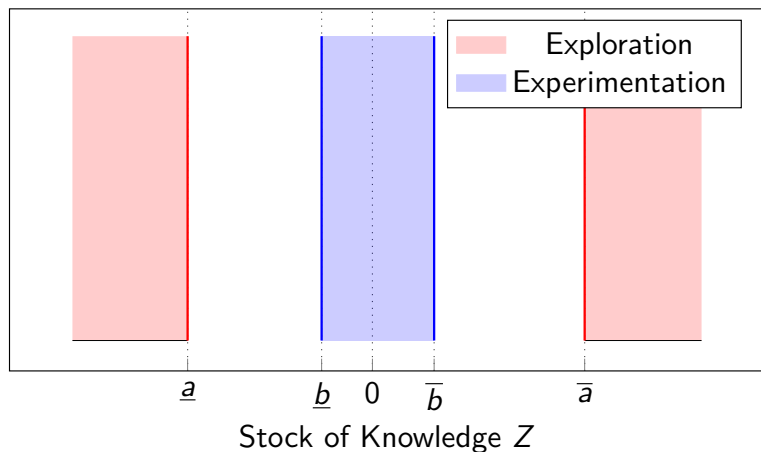
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Firm Value

We compute the intensive firm value piecewise:

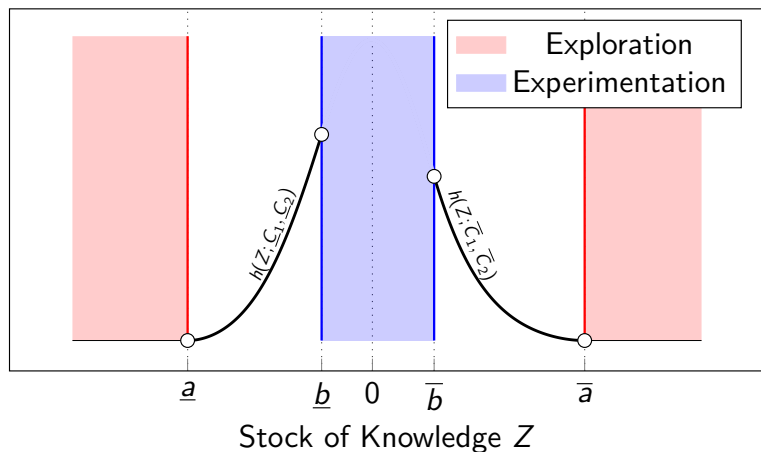
$$v(Z) =$$



Firm Value

We compute the intensive firm value piecewise when idle:

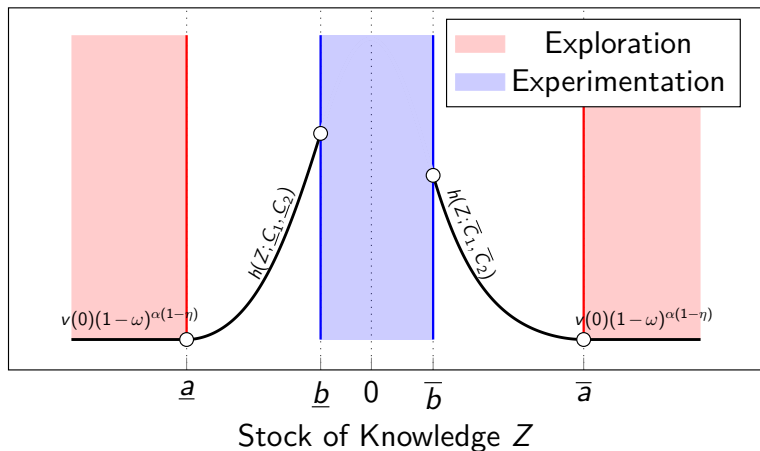
$$v(Z) = v_P(Z) + e^{g_1 Z + g_2 Z^2} (C_1 H_n(Z) + C_2 M(Z)) \equiv h(Z; C_1, C_2)$$



Firm Value

We compute the intensive firm value piecewise when exploring:

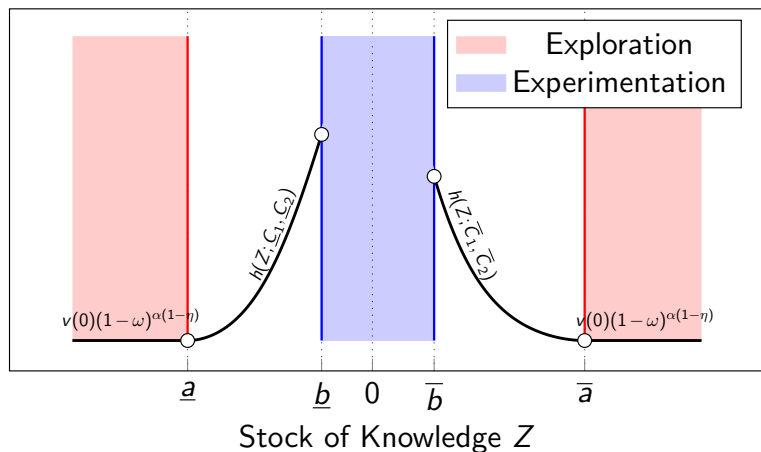
$$v(Z) = v(0)(1 - \omega)^{\alpha(1-\eta)} \equiv g(0)$$



Firm Value

Firm value solves a Bellman equation when experimenting:

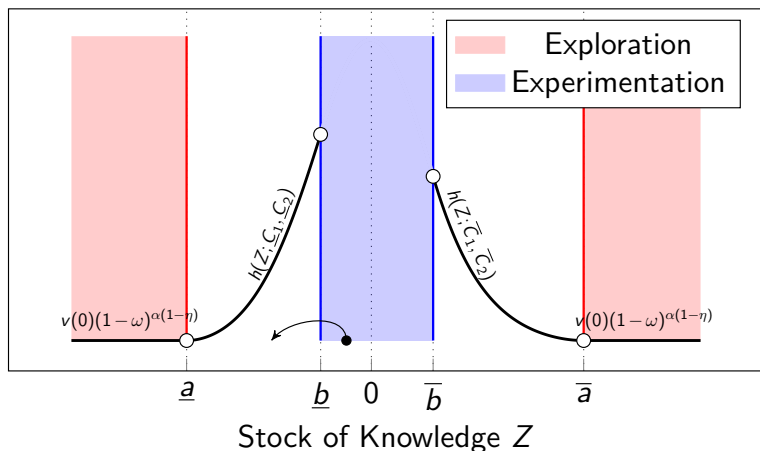
$$V(Z) = \max_i -\kappa - \gamma/2i^2$$



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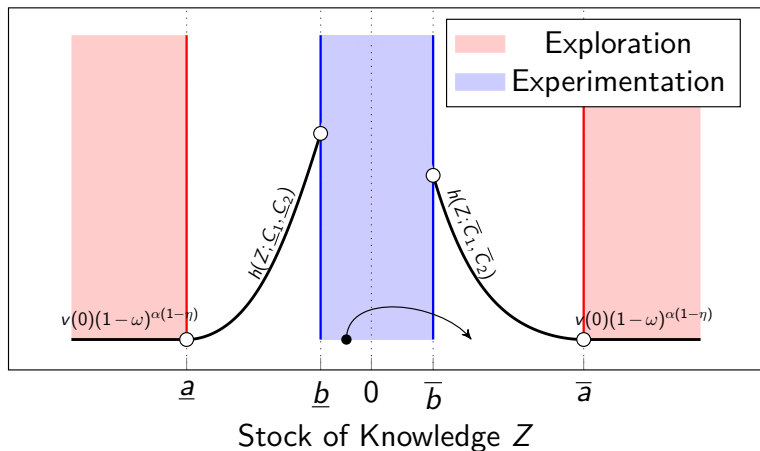
$$V(Z) = \max_i -\kappa - \gamma/2i^2 + (1 + i)^{\alpha(1-\eta)} \left(\int_{\underline{a}}^{\underline{b}} h(x; \cdot) d\Phi(x; Z) \right)$$



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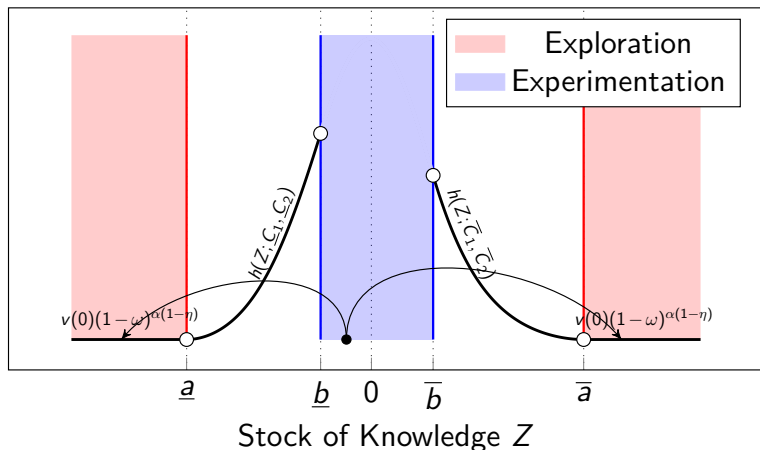
$$V(Z) = \max_i -\kappa - \gamma/2i^2 + (1+i)^{\alpha(1-\eta)} \left(\int_{\underline{a}}^{\underline{b}} h(x; \cdot) d\Phi(x; Z) + \int_{\bar{b}}^{\bar{a}} h(x; \cdot) d\Phi(x; Z) \right)$$



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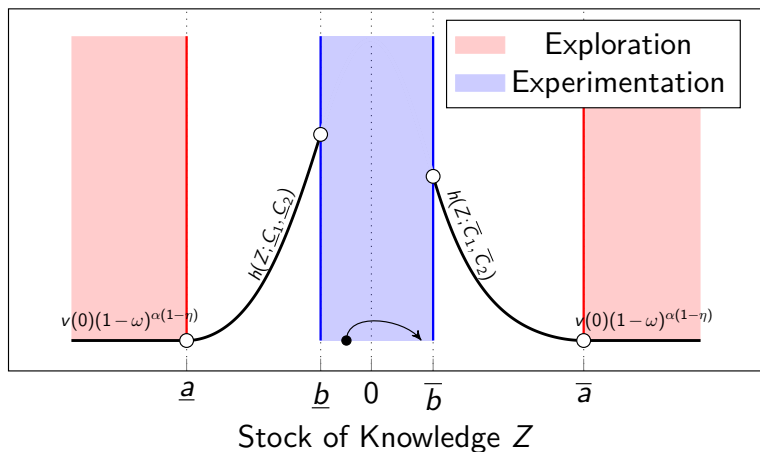
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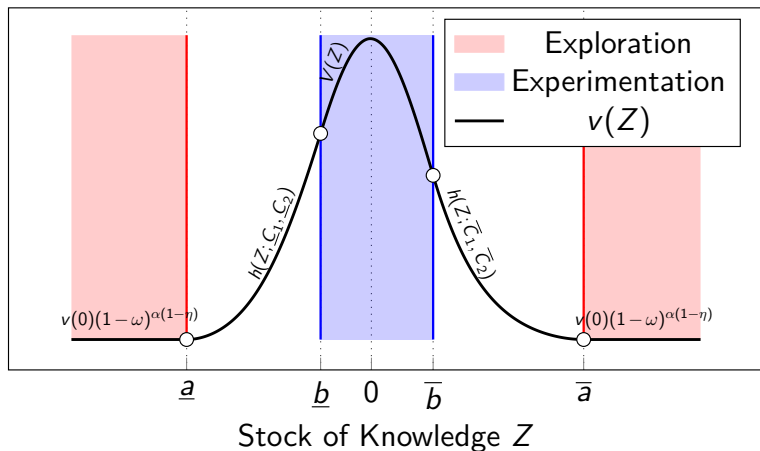
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Corporate Investment and the Knowledge Channel

FOC for the optimal knowledge-contingent investment plan is:

$$i(Z) = \frac{1}{\gamma} \left(q(Z) + \int_{\mathbb{R}} \frac{V_{\cdot}(\cdot, K+I, x)}{\Pi(\cdot, K)} \frac{d}{di} \varphi(x, i, Z) dx \right),$$

where $q(Z) \equiv \int_{\mathbb{R}} \frac{V_K(\cdot, K+I, x)}{\Pi(\cdot, K)} \varphi(x, i, Z) dx$ (Abel & Blanchard (1986))

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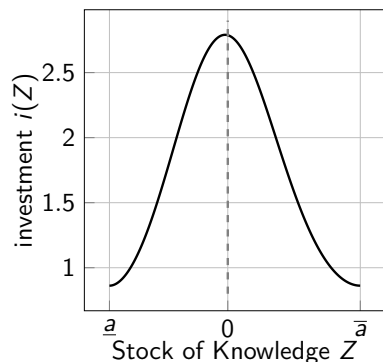
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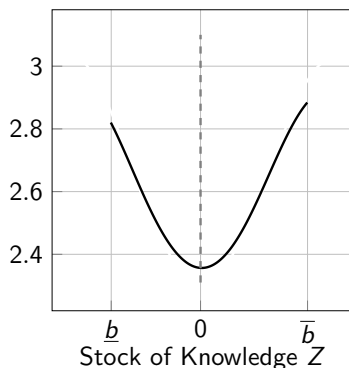
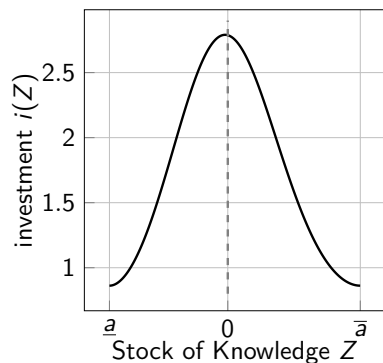
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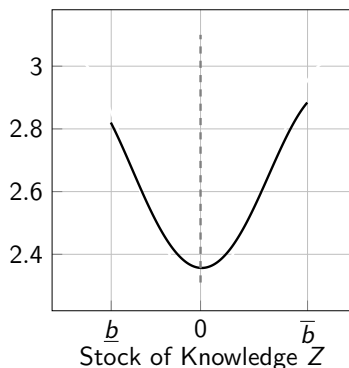
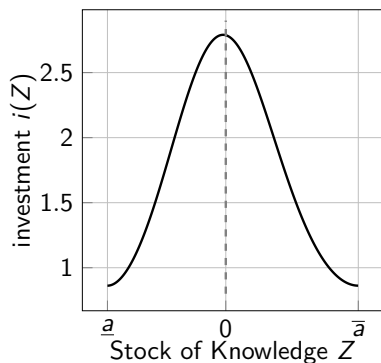


Corporate Investment and the Knowledge Channel

Use Stein's lemma to decompose knowledge channel:

$$\underbrace{\int_{\mathbb{R}} \frac{V(\cdot, K+I, x)}{\Pi(\cdot, K)} \frac{d}{di} \varphi(x, i, Z) dx}_{\text{knowledge channel } \propto \tau_S} = \frac{\tau_S}{2} (1+i)^{\alpha(1-\eta)} \left(\underbrace{\frac{Z}{\sqrt{1+\tau_S i}} \int_{\mathbb{R}} v'(x) \varphi(x, i, Z) dx}_{\text{marginal product of knowledge}} + \underbrace{\int_{\mathbb{R}} v''(x) \varphi(x, i, Z) dx}_{\text{attitude towards risk}} \right)$$

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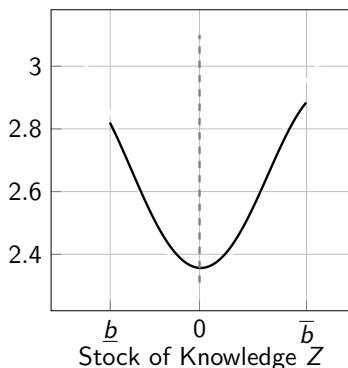
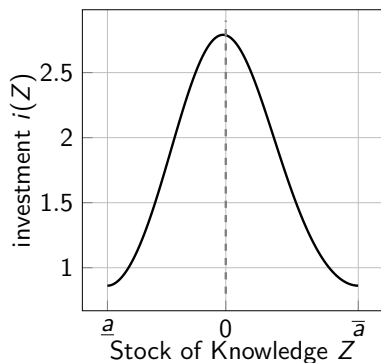


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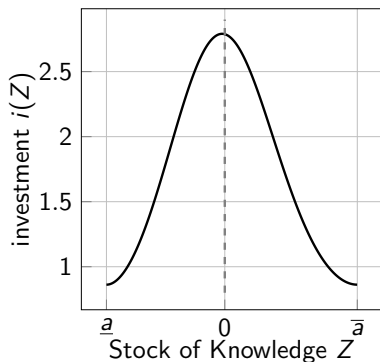


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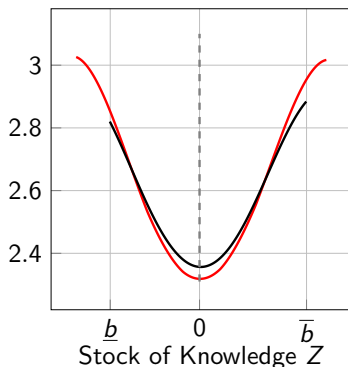
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↗ competitive pressures



Q-Investment Relation and Knowledge Cycles

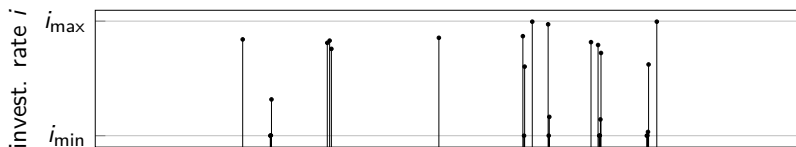
Typical investment-average $Q(Z) \equiv V(K, Z, \cdot) / \Pi(K, \cdot)$ regression:

$$i(Z_t) = \alpha_k + \beta_k Q(Z_t) + \epsilon_t, \quad |Z_t| \in \text{decile}_k, \quad k = 1, \dots, 10$$

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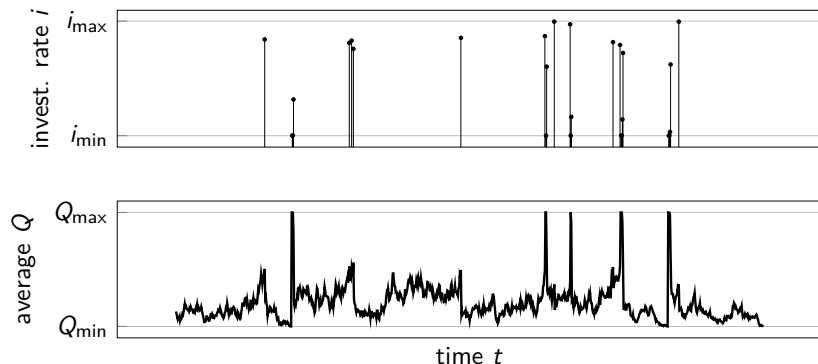
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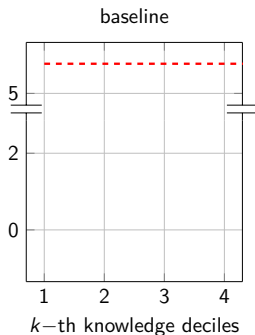
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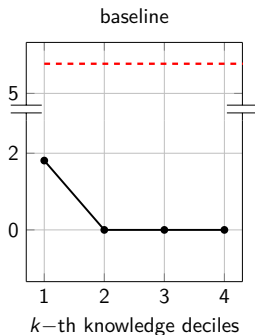
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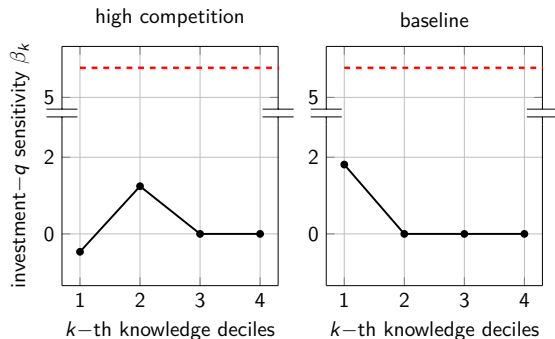
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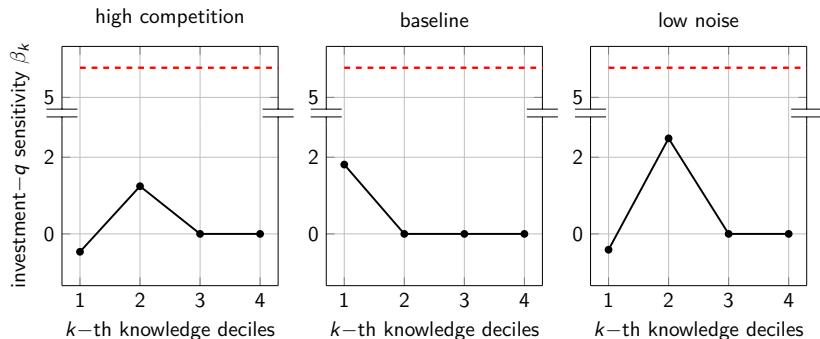
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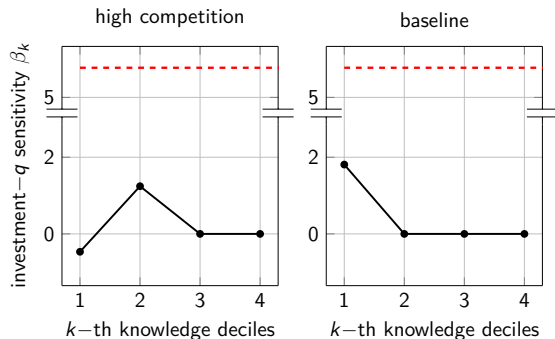
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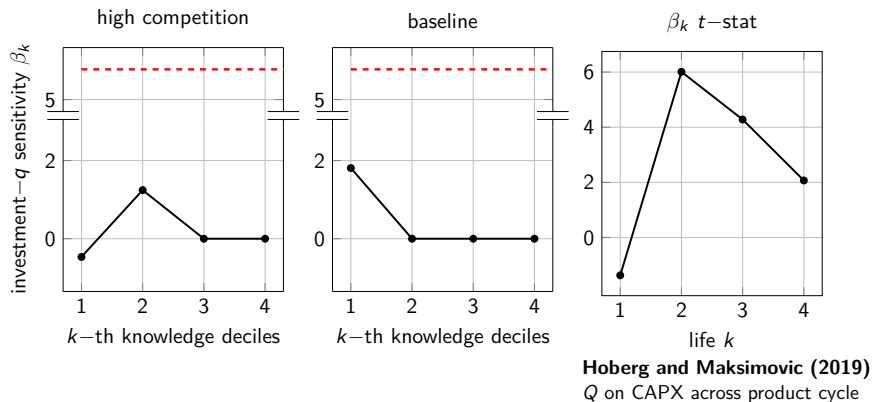
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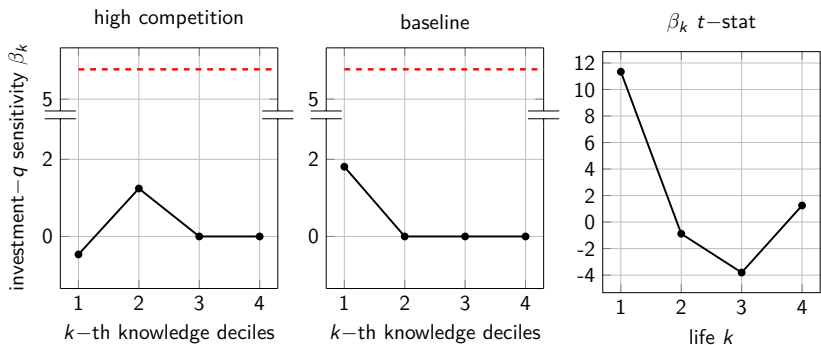
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Hoberg and Maksimovic (2019)
Q on R&D across product cycle

Ideas for testing the model

- Measuring knowledge cycles from patents' text
 - Appearance and disappearance of technological words
 - Similar to Bowen et al. (2019)
 - Exploit variation in investment (and Q) across industries
- Measuring knowledge cycles from product descriptions
 - Similar to Hoberg and Maksimovic (2019)
 - Exploit variation in investment (and Q) across firms
- Empirical relevance and economic quantification (structural?)

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