Learning from Interest Rates: Implications for Stock-Market Efficiency*

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Abstract

We analyse how rational investors use information contained in interest rates to learn about stock-market fundamentals. For that purpose, we develop a competitive noisy rational expectation equilibrium model in which the rate of interest is determined endogenously. We demonstrate that the interest rate reveals information about discount rates which helps investors extract more accurate information about cashflows from the stock's price. The strength of this mechanism and, hence price informativeness, is increasing in the rate of interest. Consequently, a lower mean bond supply (e.g., as a result of quantitative easing) leads to lower stock-price informativeness and, in turn, to a higher return volatility and price of risk. We discuss how fiscal and monetary policies, through their impact on interest rates, affect informational efficiency and other properties of the stock market. We report robust empirical evidence supporting our key prediction that price informativeness is positively related to the level of the interest rate.

Keywords: (endogenous) interest rates, informational efficiency, rational expectations, fiscal and monetary policy

JEL: G11, G14

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Interest rates play an essential role in financial markets. Foremost, they represent the rate at which investors discount risk-free as well as risky future cash flows. But, they also convey information about the economic outlook. Indeed, there exists substantial evidence that information contained in the yield curve forecasts stock-market returns, and specifically, that its slope predicts recessions.[1] In recent years, however, market participants have expressed concerns that unconventional monetary policy (i.e., quantitative easing) has distorted rates to the point that they have lost their predictive power.[2]

The purpose of this paper is to investigate such claims. Specifically, we study how investors extract information from interest rates to learn about economic fundamentals. Our analysis sheds light on how bond-market characteristics (e.g., the supply of bonds) might impair informational efficiency, that is, the ability of financial markets to aggregate and disseminate private information. More generally, it allows for a more complete understanding of the impact of government (central bank) policies on economic outcomes.

Rational expectations equilibrium (REE) models would lend themselves naturally to study such questions. However, virtually all such models assume (for tractability) that the rate of interest is exogenous, thus ruling out learning from rates. Accordingly, we develop a REE model in which the interest rate is endogenous and utilized by rational investors to update their beliefs. We then vary bond-market characteristics to study the implications of variations in monetary and fiscal policies.

Specifically, ours is a standard REE model, in the spirit of Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), but for one difference: the rate of interest is determined endogenously by supply and demand; in other words, we relax the common assumption that the bond is in perfectly elastic supply. As a consequence, the interest rate contains information in equilibrium, which rational investors attempt to extract. To allow for consumption goods rather than the bond to serve as numeraire (so that the bond price is not

[1]The literature on the informational content of the term structure is extensive. See, for example, Harvey (1988), Mishkin (1990), Estrella and Mishkin (1998), and Ang, Piazzesi, and Wei (2006).

[2]For example, in July 2018, the former chairman of the Federal Reserve, Ben Bernanke, warned that, because of "distortions" in financial markets, a yield-curve inversion might not necessarily point to a recession. Related, in September 2018, a Financial Times article entitled "How central banks distort the predictive power of the yield curve" argued that the gap between long- and short-term yields had flattened for non-fundamental reasons. Similar claims have been made about the effect of asset-purchase programmes by the European Central Bank and the Bank of Japan.

simply a normalization), we assume that investors derive utility not only from terminal but also from intermediate consumption.[3] Otherwise, the economic framework is kept as simple as possible to illustrate the economic mechanisms in the clearest possible way. In fact, our model collapses to Hellwig (1980)s classic model when learning from the interest rate is prohibited. In addition to a (real) risk-free bond with a unit payout, a risky stock is available with rational investors receiving a private signal about its random payout. Finally, noise (liquidity) traders operate in *both* markets, thus preventing the combination of the two asset prices from being perfectly revealing. Noise traders' behaviour is represented by a noisy (random) supply of assets. Shocks to this supply represent "discount rate news," whereas shocks to the stock's payout can be interpreted as "cashflow news."

To illustrate *how* rational investors can learn from interest rates, we start with a tractable version of this framework in which investors consume exclusively at the terminal date. We demonstrate that, while the bond market does not provide information about the stock's payout, it does reveal information about the stock's supply. As a result, rational investors use the information revealed by the equilibrium interest rate to update their beliefs regarding the stock's supply, which, in turn, allows them to infer more information about the stock's payout from its price. Putting it differently, the bond market conveys information about discount rates which makes stock prices more informative about cashflows.

Importantly, the strength of this mechanism is related to the level of the interest rate. Indeed, the precision of rational investors' posterior beliefs regarding the stock's supply and, hence, stock-price informativeness, are increasing in the rate of interest. Intuitively, bond-market clearing "connects" the bond and stock supplies because it guarantees that, in aggregate, investors' supply and demand (of consumption goods) balance out. A higher interest rate implies a lower bond price, and hence, a lower value of the bond's (noisy) supply, which, in turn, makes the bond a less noisy signal of the stock's supply. Because the interest rate in the economy is stochastic, this also implies that, in stark contrast to traditional REE models, price informativeness and investors' posterior precision depend on the *realization* of the state variables and, hence, are not known ex-ante. Interestingly, this mechanism

[3] In traditional REE model, the (exogenous) bond serves as numeraire, that is, the stock price is expressed in units of the bond or, putting it differently, the bond price is normalized to one.

implies that, even under a totally uninformative stock supply (i.e., its variance is infinite), the stock price provide information about the payoff (because its variance conditional on the bond signal is not infinite).

Methodologically, we are able to characterize the equilibrium in closed-form—even though both the interest rate and the stock price are non-linear functions of the state variables in equilibrium. The key idea is to stipulate ("guess") the functional form of the market-clearing conditions (which remain linear), rather than of the interest rate and the stock price themselves.

Having understood the economic mechanism at work, we switch intermediate consumption back on. Because investors save in order to smooth their consumption, and their lifetime wealth depends on their expected profits from trading the stock, itself a function of the stock's squared Sharpe ratio, the bond-market-clearing condition is no longer linear in the state variables. For this reason, we devise to a (novel) algorithm to solve for the equilibrium numerically. While the bond-market-clearing condition is considerably different in the presence of intermediate consumption, investors use information revealed by the bond market in the same way as before. That is, the bond market-clearing condition "connects" the realizations of the bond and stock supplies, so that the equilibrium rate of interest serves as a signal about the stock supply perturbed by noise from the bond supply. Consequently, investors use the information revealed by the interest rate to update their beliefs about the stock supply (discount rates), which in turn, allows them to extract more payout (cashflow)-information from the stock's price. As a result, price informativeness and investors' posterior precision depend on the interest rate. Indeed, we find again that both are increasing in the rate of interest.

We then study how characteristics of the bond market, especially, the mean of the bond supply, affect equilibrium asset prices and their informativeness. Intuitively, the interest rate is increasing in the mean bond supply as a higher supply requires a lower bond price for the market to clear. It follows that price informativeness is also increasing in the mean bond supply, an increase that can be entirely attributed to learning from interest rates. These changes in the equilibrium interest rate and price informativeness, naturally, have an impact on stock return moments. A higher price informativeness (caused by a higher mean supply) leads to less risk and therefore to lower expected excess return, return volatility and Sharpe ratio on the stock.

In the last step of our theoretical analysis, we incorporate a government into the economy. We allow for government spending and taxation as well as money. This addition allows us to relate interest rates to monetary and fiscal policies, and thus to speak to the influence of these policies on informational efficiency. We capture the usefulness of money as a medium of exchange by introducing real money balances in investors' utility function. As in the stock and bond markets, we assume that the residual supply of money is noisy. While the market-clearing condition for the bond again "connects" the realizations of the bond and stock supplies, the residual money supply injects additional noise into the bond signal.[4] Offsetting the increased noisiness of the bond signal, investors now observe another signal, namely, the inflation rate (i.e., the period-1 good's price) which also contains information about the stock supply.[5] Hence, both the rate of interest and the good's price allow investors to form more precise posterior beliefs about the discount rates (stock's supply) which, in turn, allows them to extract more cashflow information from the stock's price.

Finally, we conduct empirical tests of our central finding that stock-price informativeness increases with the level of interest rates. Using a measure of informativeness developed in the literature which matches our model closely (Bai, Philippon, and Savov (2016)), we report evidence that price informativeness rises with interest rates, as predicted by our model. The effect is economically significant: Our baseline estimate indicates that a one standard-deviation increase in interest rates leads to an increase in price informativeness of about two thirds of a standard-deviation. We confirm that our findings are statistically robust. They obtain for both nominal and real interest rates, across bond maturities ranging from one to 10 years, and for forecasting horizons of 3 and 5 years; they hold up to the inclusion of control variables and to variations in the estimation procedure (OLS, Newey-West, non-parametric).

[4] Because Ricardian Equivalence holds in our economy, the noise that ultimately blurs prices is government consumption regardless of how it is financed.

[5] In the presence of money, money, rather than the good, serves as a numéraire.

Our paper is related to three strands of the literature. First and foremost, it builds on the extensive noisy REE literature initiated by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982). Our main contribution to this literature is to endogenize the rate of interest. We show that the interest rate contains valuable information about a stock's noisy supply, and work out how investors use this information to update their beliefs about its payoff. We are not aware of other work in which the stock price and the interest rate *both* reveal information. A consequence is that price informativeness and investors' posterior precision are increasing functions of the interest rate. This property, in turn, leads to three implications that that further distinguish ours from most noisy REE models.

First, because the interest rate is stochastic, price informativeness and posterior precision are themselves stochastic, rather than deterministic as in traditional models with Gaussian shocks. Second, how much investors know about the stock market varies along the business cycle. This finding links our work to that of Kacperczyk, van Nieuwerburgh, and Veldkamp (2016) who analyse how investors knowledge depends on the state of the economy. But the mechanisms are markedly different in that this dependence stems from (exogenous) variations in risk and in its price in Kacperczyk, van Nieuwerburgh, and Veldkamp (2016), vs. from (endogenous) variations in interest rates in our model. Third, the sensitivities of the stock price with respect to the state variables (the random payout and noise) are stochastic and, hence, the stock price is a non-linear function of the state.

Our paper connects further to three sub-streams of the noisy REE literature. The first studies economies with multiple assets such as Admati (1985), Brennan and Cao (1997), Kodres and Pritsker (2002), van Nieuwerburgh and Veldkamp (2009), van Nieuwerburgh and Veldkamp (2010), or Kacperczyk, van Nieuwerburgh, and Veldkamp (2016). Though our model features two assets with informative prices, it differs distinctly from these models in that our other asset is *riskfree*. In particular, we show that the riskfree asset reveals information about the stock despite its payoff and supply being uncorrelated with those of the stock. This is in sharp contrast with Admati (1985) and the work that followed, in which, absent cross-asset correlations, nothing is to be learned from one asset about another. Note also that these model lead to a deterministic price informativeness, whereas it is stochastic in our framework. Second, through its emphasis on information about the stock's supply, our work is also related to papers such as Watanabe (2008), Ganguli and Yang (2009), Manzano and Vives (2011) and Farboodi and Veldkamp (2017). In these papers, investors receive a private and exogeneous signal (which they either purchase or are endowed with) about the stock supply. In contrast, the supply signal—also referred to as (order-)flow or discount rate information in this literature—is public and endogenous in our setup. Finally, our paper is part of the sub-stream of the literature that seeks to generalize noisy REE models and explore their robustness to assumptions (see, e.g., Barlevy and Veronesi (2000), Barlevy and Veronesi (2003), Peress (2004), Breon-Drish (2015), Banerjee and Green (2015) and Albagli, Hellwig, and Tsyvinski (2015)). Our contribution is to endogenize the interest rate in an otherwise conventional noisy REE model and identify what features survive or differ.

The second stream of research to which our paper belongs studies the importance of an *endogenous* rate of interest in asset-pricing models under *symmetric* information. Lowenstein and Willard (2006) highlight that, under the assumption of a storage technology (i.e., riskless asset) in perfectly elastic supply, aggregate consumption risk differs from exogenous fundamental risk and that this can yield misleading conclusions (e.g., with respect to the impact of noise traders or violations of the Law of One Price). Our work is distinctly different from their paper due to the presence of private information and our focus on price informativeness. Moreover, we find that the main conclusions of the traditional noisy REE literature are robust to endogenizing the risk-free rate. Instead, we illustrate that new (unexplored) mechanisms arise when the bond market clears under a fixed bond supply.

Third, our work contributes to the literature studying the impact of fiscal and monetary policy on asset prices; for example, Croce, Nguyen, and Schmid (2012) and Croce, Kung, Nguyen, and Schmid (2012). While this literature typically relies on the assumption of symmetric information, we allow for private (asymmetric) information. In so doing, we can analyse the impact of these policies on the informational efficiency of the stock market, and extrapolating our results, on the efficiency of the capital allocation. Moreover, a large literature in macroeconomics studies the impact of interest rates on efficiency in the presence of frictions in the credit market. We too focus on the link between interest rates and efficiency, but the frictions we consider operate in the stock market (asymmetric information). Empirically, our finding that informational efficiency is worse in low interest rate environments is consistent with recent evidence documenting that low interest rates (whether they are caused by demographic shifts, low productivity growth or the recent unconventional monetary policies) lead to misallocated resources (see, e.g., Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez (2017)).

The remainder of the paper is organized as follows. Section 1 introduces our main economic framework. Section 2 discusses, in a tractable version of the model, the economic mechanism through which rational investors learn from the bond market. In Section 3, we then study the full model and relate the characteristics of the bond market to equilibrium outcomes. Section 4 studies the impact of government policies and Section 5 provides empirical evidence on our key predictions. Finally, Section 6 concludes. Proofs and a description of the numerical solution approach are delegated to the Appendix.

1 A REE-Model with Bond-Market Clearing

In this section, we introduce our main economic framework. The framework differs from traditional competetive rational expectation equilibrium (REE) models, such as Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), along three key (related) dimensions. First, the rate of interest is determined endogenously. Second, rational investors learn not only from their private signals and the stock price but also from the interest rate. Third, agents consume not only in the final period, but also in the trading period. In the following, we discuss the details of the model.

Information Structure and Timing

We consider a two-period model. Figure 1 illustrates the sequence of the events. In period 1, the trading round, rational investors observe their private signals (with given precision) and equilibrium asset prices. Based on this information, they choose their portfolio holdings

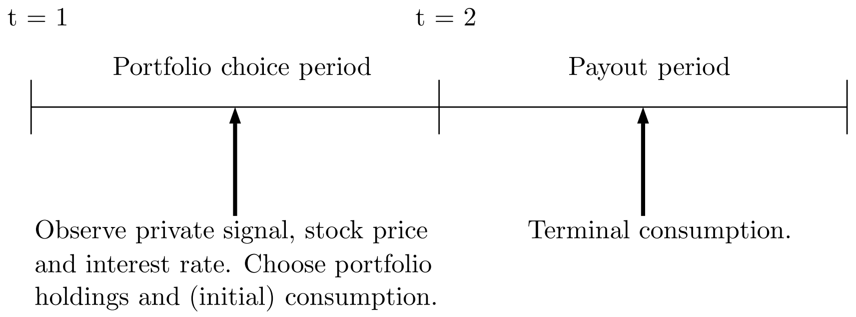

Figure 1: Timing. The figure illustrates the sequence of the events.

and (period-1) consumption (initial consumption). Asset prices are set such that financial markets clear. In period 2, the payoff period, investors simply consume the proceeds from their investments (terminal consumption). We denote rational investors' expectation and variance conditional on their time-1 information set $\mathcal{F}_i$ as $E[\,\cdot\,|\,\mathcal{F}_i]$ and $Var(\,\cdot\,|\,\mathcal{F}_i)$.

Investment Opportunities

Two financial securities are traded in competitive markets: a (real) risk-less asset (the "bond") and a risky asset (the "stock"). The consumption good that serves as numéraire, hence all prices and payoffs are denominated in units of the good. The bond has a payoff of one in period 2 and its price is denoted by P_Y.[6] Hence, the (gross) rate of interest is given by $R_f \equiv 1/P_Y$. The stock is a claim to a random payoff $\Pi \sim \mathcal{N}(E_\Pi, 1/\tau_\Pi)$, which is only observable in period 2, and its price is denoted by P_X.[7] The stock also makes a deterministic payout of Π_1 in period 1. Both assets are in inelastic (finite) supply.

Investors

There exists a continuum of atomless rational investors of mass one. At the beginning of period 1, each rational investor i receives a private signal $S_i = \Pi + \varepsilon_i$, $\varepsilon_i \sim \mathcal{N}(0, 1/\tau_\varepsilon)$ with precision τ_ε.

[6] This contrasts with traditional REE models, such as Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982), in which the (exogenous) riskless bond serves as numéraire.

[7] Throughout the paper, we use the letter τ to denote precisions (inverse of variances).

Rational investors have CARA-preferences over initial consumption $C_{i,1}$ and terminal consumption $C_{i,2}$

$$U_i(C_{i,1}, C_{i,2}) = -\frac{1}{\gamma}\, \exp\bigl(-\gamma\, C_{i,1}\bigr) + \beta\, E_1 \left[-\frac{1}{\gamma}\, \exp\bigl(-\gamma\, C_{i,2}\bigr) \,\middle|\, \mathcal{F}_i \right], \tag{1}$$

where γ denotes (absolute) risk-aversion, $\beta \in (0.1)$ denotes the rate of time preference and $\mathcal{F}_i = \{S_i, P_X, R\}$ describes investor i's time-1 information set. Each investor is endowed with a random number shares of the stock, $X_{i,0}$ and no shares of the bond. Thus, initial wealth given by $W_{i,1} = X_{i,0}\,(P_X + \Pi_1)$. While the aggregate number of endowed shares, $\int X_{i,0}\, di$, equals the residual supply of the stock, we assume that each investor's endowment is uninformative.[8]

In addition, noise (liquidity) traders operate in *both* the bond and stock markets. Their behaviour is not explicitly modelled and characterized instead by the (random and unobservable)—residual supplies of the two assets. These residual supplies should be understood as the supply of an asset minus the noise traders' demand. Formally, the residual supply of the stock and bond are represented by exogenous random variables $\theta_X \sim \mathcal{N}(E_{\theta_X}, 1/\tau_{\theta_X})$, and $\theta_Y \sim \mathcal{N}(E_{\theta_Y}, 1/\tau_{\theta_Y})$. E_{θ_X} (resp. E_{θ_Y}) and τ_{θ_X} (resp. τ_{θ_X}) denote the mean and prior precision of the stock's (resp., bond's) supply . The random variables Π, θ_X and θ_Y are assumed to be uncorrelated. Note that, in addition to the usual stock-market noise, we assume that the supply of the bond is noisy. This prevents the bond and stock prices from being jointly perfectly revealing.

Equilibrium Definition

The objective of a rational investor i is to maximize expected utility (1) subject to the following budget equations:

$$C_{i,1} + X_i\, P_X + Y_i\, P_Y = W_{i,1}, \quad \text{and} \quad C_{i,2} = X_i\, \Pi + Y_i, \tag{2}$$

[8]This rules out learning from the initial stock endowment which, otherwise, would also serve as a signal.

where X_i and Y_i denote the investor's holdings (number of shares) of the stock and the bond, respectively.

Accordingly, a rational expectations equilibrium is defined by consumption choices $\{C_{i,1}, C_{i,2}\}$, portfolio choices $\{X_i, Y_i\}$, and asset prices $\{P_X, R_f\}$ such that:

1. $\{C_{i,1}, C_{i,2}\}$ and $\{X_i, Y_i\}$ maximize investor i's expected utility (1) subject to the budget constraints (2); taking prices P_X and R_f as given.
2. Investors' expectations are rational.
3. Aggregate demand equals aggregate residual supply—in the bond and the stock market:

$$\int X_i \, di = \theta_X, \text{ and } \int Y_i \, di = \theta_Y. \tag{3}$$

It is important to highlight that, in equilibrium, *both* asset prices play a dual role. That is, each price clears its respective market but also aggregates and transmits rational investors' private information. Note also that, by Walras' law, market clearing in the bond and the stock market guarantees market clearing in the goods market in period 1: $\int C_{i,1} \, di = \theta_X \, \Pi_1$.

2 Learning from the Bond Market

In this section, we illustrate the economic mechanisms through which an endogenous risk-free rate affects the equilibrium. For that purpose, we rely on a version of our model that is designed to provide the economic intuition and allows for closed-form solutions. It differs from our main economic framework described in the preceding section only along a single dimension: Rational investors consume exclusively at the terminal date.

2.1 Optimal Portfolio Choice

In the absence of initial consumption, the objective of each rational investor i is to choose his portfolio holdings in the stock, X_i, and in the bond, Y_i, in order to maximize expected utility over terminal consumption

$$U_i(C_{i,2}) = -\frac{1}{\gamma}\, E\left[\exp\bigl(-\gamma\, C_{i,2}\bigr) \,\big|\, \mathcal{F}_i\right],$$

subject to the budget equations $X_i\, P_X + Y_i\, P_Y = W_{i,1}$ and $C_{i,2} = X_i\, \Pi + Y_i$.

The following theorem characterizes the optimal portfolio choice for arbitrary values of posterior mean $E[\Pi \,|\, \mathcal{F}_i]$ and precision $Var(\Pi \,|\, \mathcal{F}_i)$.

Theorem 1. *Conditional on the stock price P_X, the risk-free rate R_f, and an investor's posterior beliefs and endowed wealth $W_{1,i}$, the optimal stock and bond demands equal:*

$$X_i = \frac{E[\Pi \,|\, \mathcal{F}_i] - P_X\, R_f}{\gamma\, Var(\Pi \,|\, \mathcal{F}_i)}, \qquad \textit{and} \tag{4}$$

$$Y_i = R_f\, \left(W_{i,1} - \frac{E[\Pi \,|\, \mathcal{F}_i] - P_X\, R_f}{\gamma\, Var(\Pi \,|\, \mathcal{F}_i)}\, P_X\right). \tag{5}$$

The optimal demand for the stock, X_i, is described by the standard mean-variance portfolio. It is independent of the investor's initial wealth, $W_{1,i}$ and positively related to an investor's posterior mean and precision. In contrast, the optimal demand for the bond, Y_i, is a function of the investor's initial wealth and, through the stock demand, X_i, "inversely" related to her posterior beliefs. For example, all else equal, the demand for the bond is low (negative—if the investor borrows—to finance stock purchases) for an investor who is optimistic regarding the stock's future payoff.

2.2 Equilibrium

A novelty of our framework is that the bond market clears. This has important implications for rational investors' learning and, hence, financial-market equilibrium.

Setting initial consumption to zero in the period-1 budget equation (2), substituting in the expression for initial wealth and aggregating across rational traders yields the bond market-clearing equation $R_f\, \theta_X\, \Pi_1 = \theta_Y$. Thus, this equation "connects" asset supplies in the bond and stock markets. Hence, the riskfree rate contains information about the stock

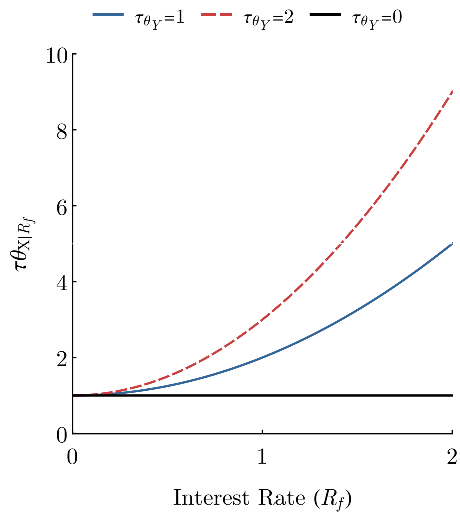

Figure 2: Posterior Precision of Stock-Market Supply. The figure shows the rational investors' posterior precision for the stock's supply, $\tau_{\theta_X|R_f}$, given in (7), as function of the rate of interest R_f—for three levels of bond-supply informativeness, τ_{θ_Y}.

market supply. Rewriting the bond-market clearing equation as $0 = \theta_X - \frac{\theta_Y}{R_f\,\Pi_1}$ demonstrates that it serves as a signal about θ_X with noise $\frac{\Theta_Y}{R_f\,\Pi_1}$. Consequently, rational investors use the information revealed by the equilibrium interest rate, R_f, to update their prior beliefs regarding the stock market supply. That will help them extract more information from the stock's price about the stock's payoff. The following lemma describes the resulting distribution of the stock's supply conditional on observing the riskfree rate.

Lemma 1. *The distribution of the stock supply, Θ_X, conditional on the equilibrium interest rate, R_f, is characterized by*

$$E_{\theta_X|R_f} \equiv E\left[\theta_X \mid R_f\right] = \frac{\tau_{\theta_X}}{\tau_{\theta_X|R_f}}\,E_{\theta_X} + \frac{R_f^2\,\Pi_1^2\,\tau_{\theta_Y}}{\tau_{\theta_X|R_f}}\,\frac{E_{\theta_Y}}{R_f\,\Pi_1};\ \textit{and} \tag{6}$$

$$\tau_{\theta_X|R_f} \equiv Var(\theta_X \mid R_f)^{-1} = \tau_{\theta_X} + R_f^2\,\Pi_1^2\,\tau_{\theta_Y}. \tag{7}$$

Intuitively, rational investors combine their prior beliefs with the signal provided by bond-market clearing to form "posterior" beliefs regarding the (unobservable) supply in the stock market. Using Bayesian updating, the posterior mean of the stock-market supply, $E_{\theta_X|R_f}$, in (7) is a precision-weighted average of the prior mean (E_{θ_X}) and the mean conditional on the bond-market signal ($\frac{E_{\theta_Y}}{R_f \Pi_1}$). Similarly, the posterior precision, $\tau_{\theta_X|R_f}$, in (6) is the sum of the prior precision (τ_{θ_X}) and the precision of the bond-market signal ($R_f^2 \Pi_1^2 \tau_{\theta_Y}$). An important property is that the posterior precision, $\tau_{\theta_X|R_f}$, is increasing in R_f^2, as Figure 2 illustrates. Intuitively, a higher value of the squared interest rate indicates more "extreme" realizations of the supply in the bond and the stock market.[9] Because such outcomes are less likely, that is, their joint likelihood is small, the likelihood ratio is more informative and, hence, investors can draw more precise inferences about the underlying stock-market supply. The figure also illustrates that this effect is stronger, the higher the precision in the bond market, τ_{Θ_Y}, is. In particular, only if the bond-market supply is completely uninformative (i.e., $\tau_{\Theta_Y} = 0$), does the bond-market signal not provide any information. In that case, the conditional distribution of stock market supply reduces to the prior distribution.

Aggregating the rational investors' asset demands, imposing market-clearing in both markets (3) and solving for the equilibrium asset prices, yields the following characterization of the equilibrium.

Theorem 2. *There exists a unique [linear] rational expectations equilibrium, characterized by the following asset prices:*

$$R_f = \frac{\Theta_Y}{\Pi_1 \, \Theta_X}; \quad \textit{and} \tag{8}$$

$$R_f \, P_X = \left(\frac{\tau_\Pi}{\tau} \, E_\Pi + \frac{t \, \tau_\epsilon \, \tau_{\theta_X|R}}{\tau} \, E_{\theta_X|R} \right) + \frac{\tau_\epsilon \, \left(1 + t^2 \, \tau_\epsilon \, \tau_{\theta_X|R}\right)}{\tau} \left(\Pi - \frac{1}{t \, \tau_\epsilon} \, \theta_X \right). \tag{9}$$

The interest rate R_f in (8) is a *non-linear* function of the realized stock market and bond market supply and, thus, stochastic.[10] It is increasing in the (residual) bond supply, θ_Y. Intuitively, a higher supply requires a lower bond price for the market to clear and,

[9] Formally, it indicates a bigger squared ratio of the realized supply in the bond market to the realized supply in the stock market.

[10] The gross interest rate R_f can be negative in this framework. It does not, however, lead to arbitrage opportunities. Indeed, negative rates are caused by the fact that investors have a preference over terminal consumption only and, consequently, the interest rate is not determined by marginal utilities. In Section 3, we demonstrate that allowing for initial consumption (in which the gross interest rate is always positive) does not change our main results.

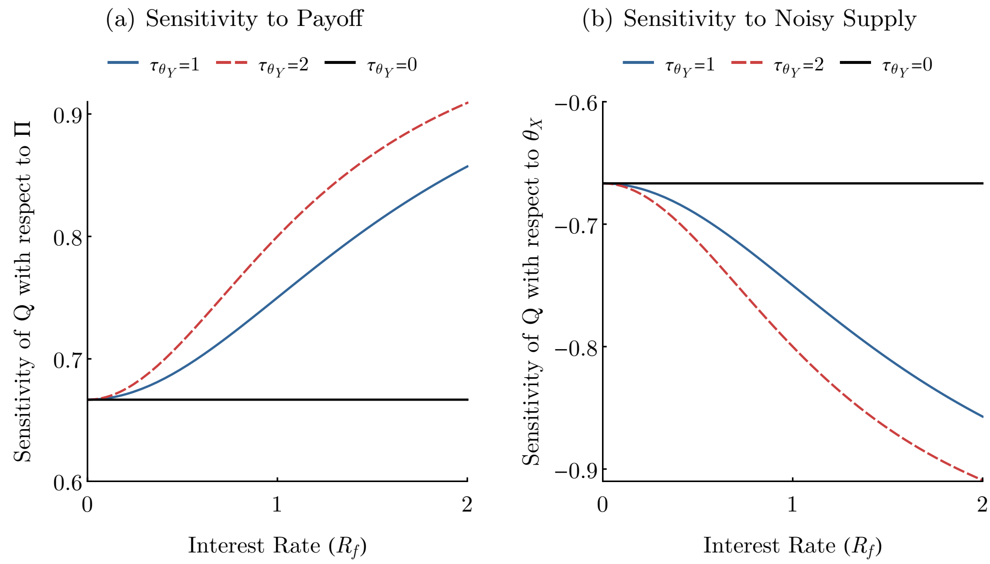

Figure 3: Price Ratio Sensitivities. The figure shows the sensitivity of the price ratio, $Q \equiv R_f P_X$, with respect to the stock's payoff Π (Panel A) and the stock's supply θ_X (Panel B) , as function of the rate of interest R_f—for three levels of bond-supply informativeness, τ_{θ_Y}.

hence, a higher interest rate. Conversely, the interest rate is declining in the (residual) supply, θ_X, and initial payout of the stock, Π_1. The reason is that an increase in either the supply or the payout of the stock increases the investors' aggregate endowment, that is, the number of available goods. Because these serve as the numéraire, the bond price increases and the interest rate drops. Putting it differently, a higher aggregate endowment increases the demand for bonds since this endowment must be saved (traders only consumed in the second period).

The characterization of the equilibrium price ratio $R_f\, P_X$ in (9) has the familiar structure of, for example, Hellwig (1980) and Verrecchia (1982). However, it is important to highlight that in our framework the equilibrium price ratio features the *posterior* mean and precision of the stock supply, $E_{\theta_X|R}$ and $\tau_{\theta_X|R}$; instead of (traditionally) its prior mean and precision. Because the interest rate R_f is stochastic, both the posterior mean (6) and posterior precision (7) are usually also stochastic and, hence, depend on the realization of the supplies in both financial markets.[11]

[11] The exception is the case in which the bond-market supply is uninformative. Recall that in this case, the posterior distribution of stock supply reduces to its prior distribution. As a result, the equilibrium price ratio, $R_f\, P_X$, coincides with that in Hellwig (1980). However, the interest rate will remain stochastic in that special case, so that equilibrium is not identical to Hellwig (1980).

This observation has important implications. First, all coefficients of the equilibrium price ratio or, equivalently, the equilibrium stock price, are *stochastic*. Consequently, the sensitivity of the equilibrium stock price with respect to the underlying payoff, Π, and the residual stock supply, θ_X, depend on the realization of the state. This is illustrated in Panels A and B of Figure 3 which show the sensitivity of the price ratio with respect to the payoff and the stock-market supply, respectively. In particular, both sensitivities are increasing (in absolute value) with the interest rate R_f because this implies more precise beliefs about the stock supply. Moreover, the magnitude of the effect is increasing in the precision of the bond supply; with both sensitivities being constant, as in Hellwig (1980), if the bond-market is uninformative ($\tau_{\theta_Y} = 0$).

Second, the equilibrium price ratio and the equilibrium stock price are *non-linear* functions of the stock's payout Π, its supply Θ_X and, implicitly, the bond supply Θ_Y. Both results are in stark contrast to the traditional framework with an exogenous interest rate in which the sensitivities are constant and, thus, the equilibrium stock price is a linear function of the state variables.

Methodologically, we are able to characterize the equilibrium in closed-form—even though both the equilibrium interest rate and the equilibrium stock price are non-linear functions of the state variables (Π, θ_X and θ_Y). As discussed in detail in the Appendix, the key idea is to stipulate ("guess") the functional form of the market-clearing conditions (which remain linear); instead of stipulating [guessing] the functional form of the interest rate and the stock price (which are not linear). Intuitively, this means that rational investors extract information from the market-clearing conditions rather than from prices themselves. This makes it possible to solve the rational investors' inference problem in closed-form and, in turn, obtain closed-form expressions for equilibrium asset prices.

2.3 Equilibrium Price Informativeness

Finally, we compute the posterior precision of investors' beliefs.

Theorem 3. *The posterior precision of the rational investors' beliefs regarding the stock's payoff, τ, is given by:*

$$\tau \equiv Var\left(\Pi \,|\, \mathcal{F}_i\right)^{-1} = \tau_\Pi + \tau_\varepsilon + t^2 \tau_\varepsilon^2 \, \tau_{\theta_X | R_f}. \tag{10}$$

The posterior precision in our framework has the same form as in Hellwig (1980) and is made up of three components: (i) the precision of the investors' prior beliefs, (ii) the precision of their private signal, and (iii) the precision of the stock-price signal ("price informativeness"). In particular, the posterior precision is increasing in the prior precision, the precision of private information and the prior precision of the stock-market supply and in investors' risk-tolerance.

However, similar to the equilibrium price function, it differs along one key dimension. That is, instead of the investors' prior precision regarding the stock supply, τ_{Θ_X}, the public-signal component (third term in (10)) features the posterior precision, $\tau_{\theta_X | R_f}$. As a result, the precision of the stock-price signal and, hence, investors' posterior precision is, in general, higher than in Hellwig (1980). These increases in precision can be exclusively attributed to learning from the bond market, that is, to the extra information investors have regarding the stock supply. Note that the "signal-to-noise ratio" of the equilibrium price function is the same as in Hellwig (1980) and unaffected by market-clearing in the bond market.[12]

It is important to highlight that the precision of the stock-price signal and, in turn, the posterior precision in (10) depend on R_f^2. In particular, one of the key prediction of our framework is that *stock-price informativeness is increasing in the squared (absolute) level of the interest rate.* That is, as discussed above, a higher squared value of the interest rate allows rational investors to more precisely infer the stocks supply because it requires more extreme and, hence, less likely realizations of the state variables Θ_X and Θ_Y (see also (8)), and hence to extract more information from the stock's price about the stock's payoff. The dependence on the interest rate also implies that, in stark contrast to traditional REE

[12]Formally, the stock's signal-to-noise ratio is defined as the ratio of the sensitivity of the stock price with respect to the payoff Π to the sensitivity of the stock price with respect to the stock supply Θ_X. Given the expression in (9), this ratio reduces to $t\,\tau_\varepsilon$, as in Hellwig (1980) and Verrecchia (1982).

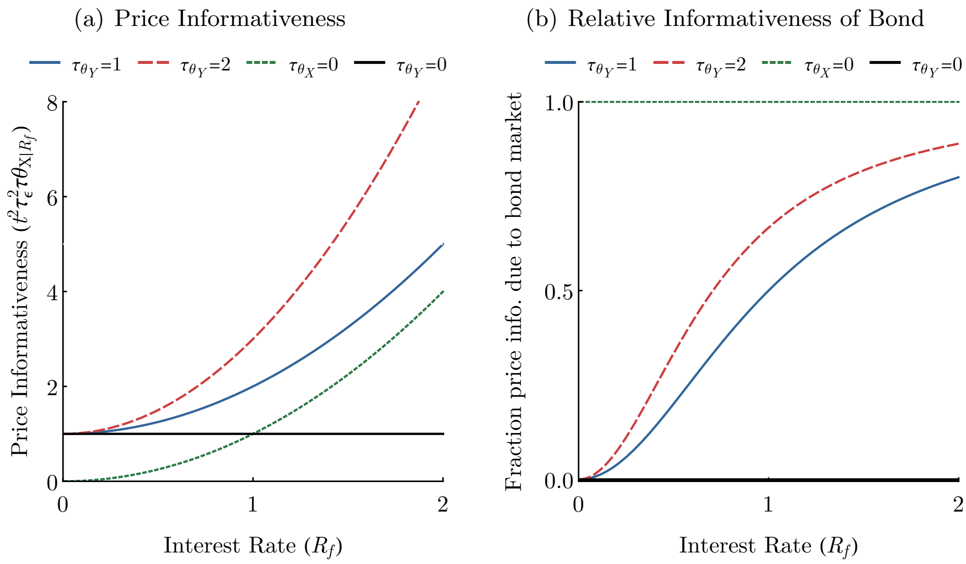

Figure 4: Price Informativeness. The figure illustrates the informativeness of the asset prices. Panel A shows the price informativeness of the public stock price and Panel B shows the fraction of the price informativeness that can be attributed to the rational investors' posterior beliefs' regarding the stock's supply: Both are shown as functions of the rate of interest R_f—for three levels of bond-supply informativeness, τ_{θ_Y} and the case in which the stock' supply is completely uninformative ($\tau_{\theta_X} = 0$.

models with Gaussian shocks, price informativeness and the posterior precision depend on the *realization* of the state variables Θ_X and Θ_Y and, hence, are not known ex-ante.

Panel A of Figure 4 illustrates this effect. In particular, it shows that price informativeness increases in the interest rate and that this effects is stronger for more precise beliefs about the bond supply (i.e., higher τ_{Θ_Y}) because this allows investors to form more precise posterior beliefs about the stock supply.[13] Panel A of Figure 4 also highlights two interesting limiting cases. First, if the bond supply is uninformative ($\tau_{\Theta_Y} = 0$), the precision of the stock? signal and, hence, the posterior precision are constant (and the same as in Hellwig (1980)). This is because in this case the bond- signal cannot be used to form more precise beliefs about the stocks supply; that is, $\tau_{\theta_X|R_f} = \tau_{\theta_X}$. Second, even if the stock supply is completely uninformative ($\tau_{\Theta_X} = 0$), the stock signal provides information about the the stock's payoff, that is, $\tau_{\theta_X|R_f} \geq 0$. Intuitively, even if the stock supply is uninformative, the posterior precision of the stock supply is informative (as long as $\tau_{\Theta_Y} > 0$) which, in turn,

[13]In that regard, the figure is reminiscent of Figure 2 which shows the posterior precision of the stock-market supply.

allows investors to infer information about the stock's payoff. That is not possible in the Hellwig (1980).

Finally, Panel B of Figure 4 reports the fraction of the price informativeness that can be attributed to the bond-market signal (relative to the overall price informativeness). It illustrates the importance of learning from the bond-market relative to learning from the stock-market. As expected, the importance of bond-market learning increases with the interest rate and the precision of the bond-market supply because these imply a more precise bond-market signal. Interestingly, the fraction of price informativeness resulting from bond-market learning is often sizeable and, for some values of the interest rate and bond-supply precision, bond-market learning accounts for the bulk of the price informativeness. Naturally, in the two limiting cases, the relative contribution of the bond-market signal is zero ($\tau_{\Theta_Y} = 0$) or one ($\tau_{\Theta_X} = 0$).

3 Rational Expectations Equilibrium with an Endogenous Interest Rate

Having established the main economic mechanisms, we now study the [full-fledged] REE-framework introduced in Section 1—in the presence of initial consumption. We then discuss the impact of the characteristics of the bond market, namely, the mean and precision of the bond supply, on the equilibrium outcome.

3.1 Optimal Consumption and Portfolio Choice

The objective of a rational investor i is to choose consumption in periods 1 and 2, $C_{i,1}$ and $C_{i,2}$, as well as holdings of the bond and the stock, Y_i and X_i, to maximize expected utility (1) subject to the budget constraints displayed in (2). Substituting out $C_{i,2}$ and solving the first-order conditions, yields the following optimal consumption and portfolio choices in period $t = 1$.

Theorem 4. *Conditional on a rational investor's posterior beliefs, the endowed wealth $W_{1,i}$, the stock price P_X and the risk-free rate R_f, the optimal consumption and portfolio choices* *in period $t = 1$ are given by:*

$$C_{i,1} = \frac{1}{1+R_f}\left(-\frac{\log\left(\beta\,R_f\right)}{\gamma} + R_f\,W_{i,1} + \frac{1}{2\,\gamma}\,\frac{\left(E\left[\Pi\,|\,\mathcal{F}_i\right] - R_f\,P_X\right)^2}{Var\left(\Pi\,|\,\mathcal{F}_i\right)}\right); \tag{11}$$

$$X_i = \frac{E[\Pi\,|\,\mathcal{F}_i] - P_X\,R_f}{\gamma\,Var(\Pi\,|\,\mathcal{F}_i)}; \qquad \textit{and} \tag{12}$$

$$Y_i = R_f\left(W_{i,1} - C_{i,1} - X_i P_X\right). \tag{13}$$

The investor's optimal consumption (11) is a function of her expected lifetime wealth, which equals the sum of her initial endowment invested at the riskless rate and of her expected trading profits on the stock (i.e., Sharpe ratio on the stock), and, naturally, depends on the investor's desire to smooth consumption throughout time (β) as well as across states (γ). The optimal demand for the stock (12) is the same as in the setting without initial consumption and given by the standard mean-variance portfolio under CARA-utility(see (4)). In contrast, the optimal demand for the bond (13) differs substantially. In particular, while the bond demand (5) in the absence of initial consumption was simply given by the "residual wealth" of an investor, it now also depends on initial consumption $C_{i,1}$ and, hence, on expected trading profits.

3.2 Equilibrium

Aggregating the rational investors' demand for the bond and stock and imposing market-clearing (3), yields the following conditions:

$$P_X\,R_f = \int E[\Pi\,|\,\mathcal{F}_i]\,di + \frac{\gamma\,\theta_X}{\tau}, \tag{14}$$

$$\frac{1}{\gamma}\log(\beta\,R_f) = \frac{1}{2}\int\frac{\left(E\left[\Pi\,|\,\mathcal{F}_i\right] - R_f\,P_X\right)^2}{\gamma\,Var\left(\Pi\,|\,\mathcal{F}_i\right)}di + \left(R_f\,P_X - \Pi_1\right)\theta_X + \left(1 + \frac{1}{R_f}\right)\theta_Y, \tag{15}$$

where $\tau \equiv Var(\Pi\,|\,\mathcal{F}_i)^{-1}$ denotes the rational investors' homogeneous posterior precision.

Together with the expressions for the rational investors' posterior beliefs, these two market-clearing conditions characterize the equilibrium in the economy. The market-clearing condition for the stock (14) has the traditional form and, formally, coincides with that in the model without initial consumption. In contrast, the market-clearing condition in the bond (15) differs substantially from the case without initial consumption. In particular, it now contains the speculative profits from trading the stock (first two terms on the right-hand side). Because of this term, the bond-market-clearing condition is no longer linear in the state variables (Π, θ_X, and θ_Y). As a result, it is not possible to identify the equilibrium in closed-form because the rational investors' inference problem involves non-linear functions. For example, it is not even possible to solve expression (14) for the price ratio $R_f P_X$ because the investors' conditional expectations $E[\Pi \mid \mathcal{F}_i]$ are not explicitly available. Consequently, we rely on a (novel) numerical algorithm to obtain the equilibrium. The algorithm is described in detail in the Appendix.

We highlight that, while the bond-market-clearing condition is considerably different in the presence of initial consumption, rational investors use information revealed by the bond market in the same way as before. That is, market-clearing in the bond-market "connects" the realizations of the residual bond supply, θ_Y, with that of the stock supply, θ_X. Consequently, rational investors can use the information revealed by the interest rate R_f to form posterior beliefs about the stocks supply which, in turn, affects their posterior beliefs regarding the stocks payout Π.

As a result, price informativeness and, hence, the rational investors' posterior precision depend on the interest rate R_f—as in the model without initial consumption. This is illustrated in Figure 5 (which is reminiscent of Figure 4). As shown in Panel A, in general, price informativeness is increasing in the rate of interest and this effect strengthens with the precision of the bond supply τ_{θ_X}. Only if the bond-market supply is uninformative ($\tau_{\theta_Y} = 0$), is price informativeness independent of the interest rate. Moreover, as pointed out earlier, the stock's price can convey information even if the stock's supply is uninformative ($\tau_{\theta_X} = 0$) because, conditional on the rate of interest, it is informative (as long as $\tau_{\theta_Y} \neq 0$). Panel B illustrates that similar to the model without initial consumption, a substantial fraction of the price informativeness can be attributed to the more precise beliefs about the stock's supply; as obtained from the bond market.

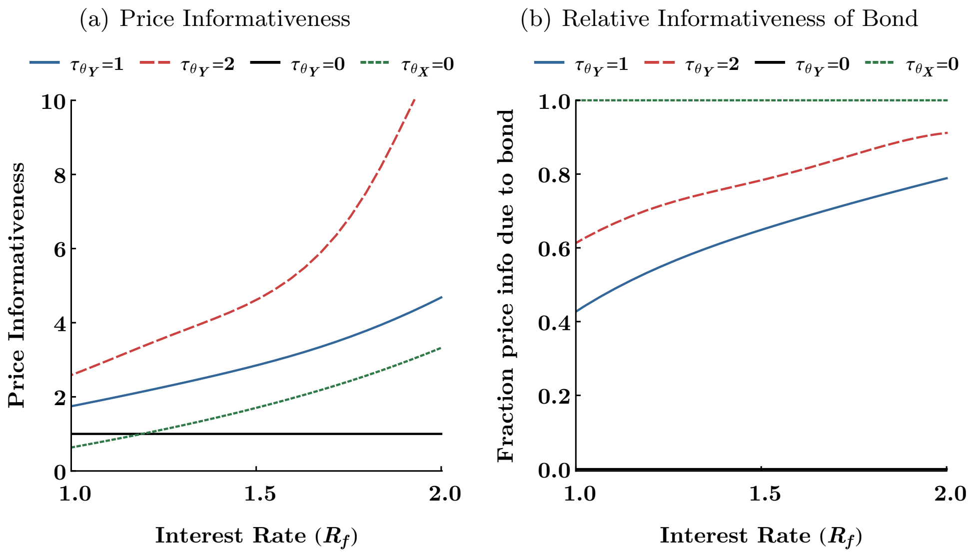

Figure 5: Price Informativeness. The figure illustrates the informativeness of the asset prices in the model with initial consumption. Panel A shows the price informativeness of the public stock price and Panel B shows the fraction of the price informativeness that can be attributed to the rational investors' posterior beliefs' regarding the stock's supply: Both are shown as functions of the rate of interest R_f— for three levels of bond-supply informativeness, τ_{θ_Y} and the case in which the stock' supply is completely uninformative ($\tau_{\theta_X} = 0$.

Finally, note that while it is not possible to obtain the equilibrium rate of interest analytically, it is straightforward to show that the gross interest rate in this setting is always positive. Intuitively, one can use each investor's first-order condition for optimal consumption $C_{i,1}$ to show that the equilibrium interest rate is pinned down by marginal utilities of periods 1 and 2. Formally, it has to fulfill

$$R_f = \frac{1}{\beta} \frac{\exp\big(-\gamma\, C_{i,1}\big)}{E\big[\exp\big(-\gamma\, C_{i,2}\big) \,|\, \mathcal{F}_i\big]} > 0.$$

That is, at the optimum, the difference between agent i's certainty equivalent of period two consumption $E[C_{i,2}|\mathcal{F}_i] - (\gamma/2)Var(C_{i,2}|\mathcal{F}_i)$ and period-1 consumption $C_{i,1}$ equals $(1/\gamma)\log(\beta\, R_f)$.

3.3 Bond-Market Characteristics and Equilibrium

The main objective of this paper is to understand the impact of the bond market on equilibrium. Accordingly, we now study how characteristics of the bond market, namely, the

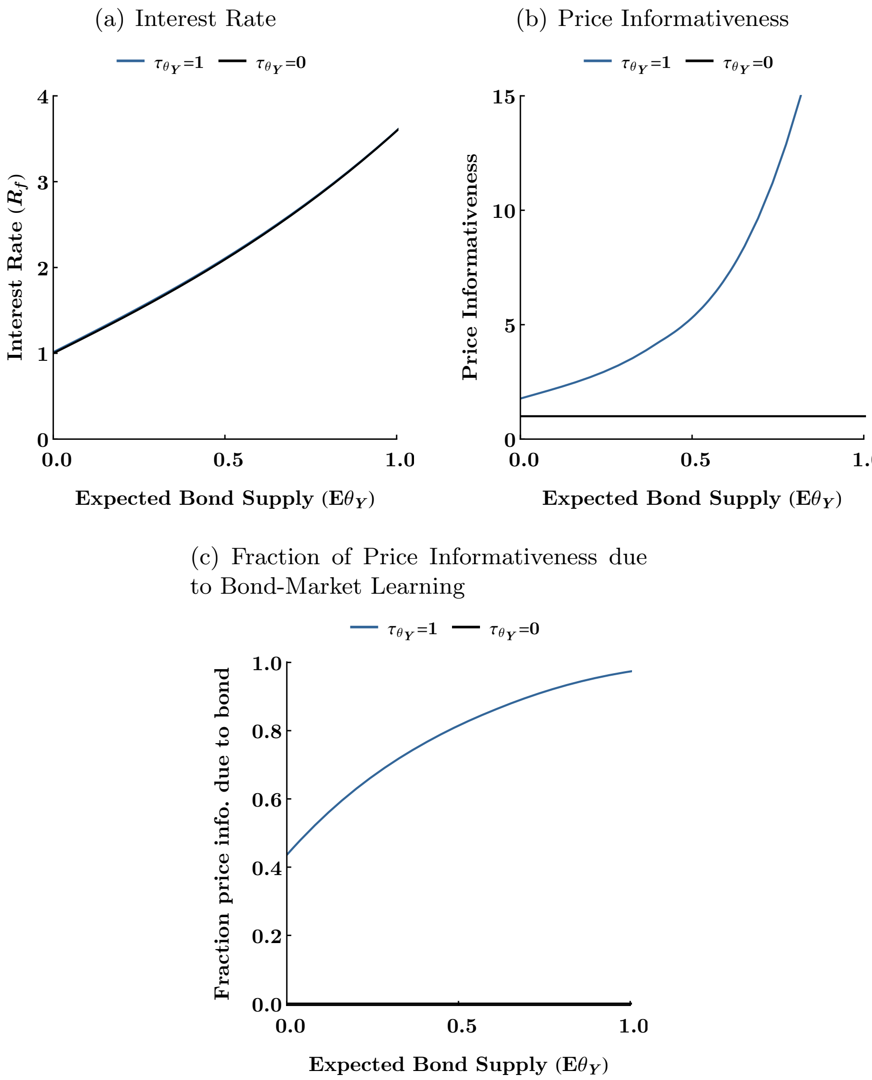

Figure 6: Impact of Bond-Market Characteristics on Price Informativeness. The figure shows how variations in the mean supply of the bond, E_{θ_Y}, and its precision, τ_{θ_Y} affect the equilibrium rate of interest and price informativeness. In particular, Panel A plots the interest rate, Panel B plots the price informativeness of the public stock price and Panel C shows the fraction of the price informativeness that can be attributed to the rational investors' posterior beliefs' regarding the stock's supply.

mean and the precision of the bond supply, shape equilibrium asset prices and their informativeness.

Figure 6 illustrates the impact of the mean, E_{θ_Y} and precision, τ_{θ_Y}, of the bond supply on the interest rate and the informativeness of the stock price in equilibrium. Panel A demonstrates that the interest rate is increasing in the mean bond supply—independently of its precision. Intuitively, a higher bond supply requires a lower bond price, and therefore a higher rate of interest for the market clear.

Panel B shows that price informativeness is also increasing in the mean bond supply, provided that the bond market is informative ($\tau_{\theta_Y} > 0$). This is driven by the increase in the rate of interest. That is, as illustrated in Figure 5 and discussed in Section 3.2, an increase in the interest rate implies that rational investors form more precise posterior beliefs about the stock's supply and, hence, better infer the stock's payout from its price. Consequently, given that the accuracy of the other sources of information (priors and private signals) is unchanged, the fraction of price informativeness attributable to bond-market learning (i.e., more precise posterior beliefs about the stock's supply) increases in the supply in the bond market. This is illustrated in Panel C of Figure 6. Only if the bond market is uninformative ($\tau_{\theta_Y} = 0$), is this effect absent and price informativeness constant(Panel B).

Naturally, these changes in the interest rate and price informativeness have an impact on the equilibrium stock price and the corresponding return moments, as illustrated in Figure 7. As shown in Panel A, the stock price decreases in the bond supply. All else equal, a higher rate of interest implies a higher discount rate and, in turn, a lower stock price. This effect is most easily seen in the case of an uninformative bond market $\tau_{\theta_Y} = 0$: an increase in the bond supply leads to an increase in the interest rate, but no change in price informativeness (see Panels A and B of Figure 6). However, if the bond market is informative ($\tau_{\theta_Y} > 0$), then there exists an offsetting effect: as the supply of the bond increases, the informativeness of the stock price rises, which in turn reduces the payoff-risk borne by investors. As a result, risk-averse investors demand a smaller discount and the price goes up. This effect can be visualized by comparing the stock price for the cases of an informative and uninformative bond market in Panel A of Figure 7. As the expected supply in the bond market increases, the gap in price informativeness across the two cases widens, pushing stock price curve further apart. This effect is also illustrated in Panel B and D which display, respectively, the asset price ratio, $P_X/P_Y = P_X R_f$ and the stock's expected excess return.

In response to an increase in informativeness, the stock's risk premium drops (Panel C), and the price of the stock declines less than does the price of the bond, leading to a reduction in their ratio (Panel B). In contrast, if the bond market is uninformative, then both the

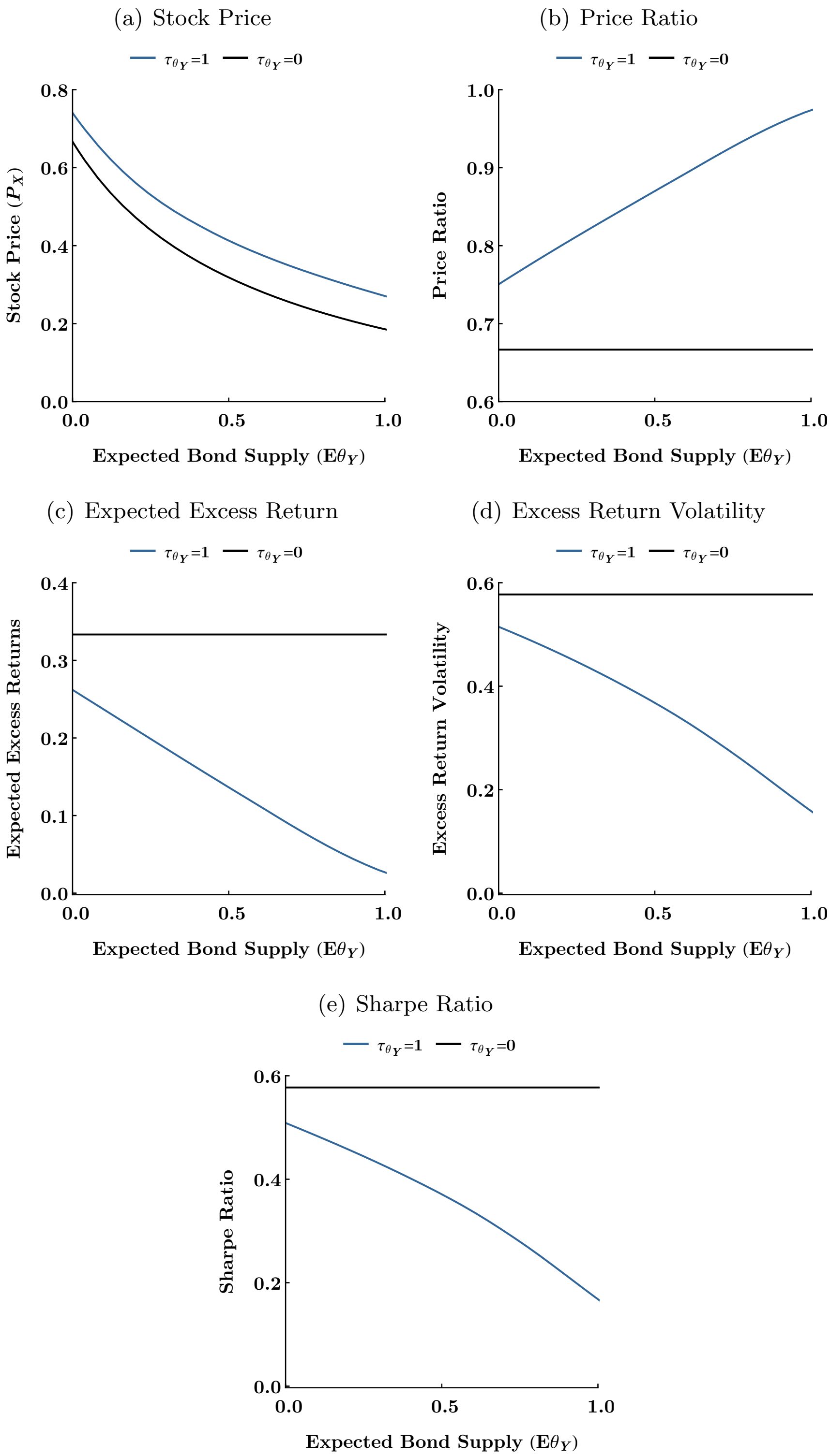

Figure 7: Impact of Bond-Market Characteristics on Asset Prices. The figure shows how variations in the mean supply of the bond, E_{θ_Y}, and its precision, τ_{θ_Y} affect equilibrium asset prices. In particular, Panel A shows the stock price, Panel B plots the price ratio, Panel C shows the expected excess return, Panel D illustrates the excess return volatility and, finally, Panel E shows the Sharpe ratio.

ratio and the premium are constant. Similarly, higher price informativeness implies that the stock price tracks the payoff more closely, thereby reducing return volatility (Panels D). Panel E illustrates that the Sharpe ratio declines with the mean supply in the bond market, that is, the decline in return volatility is more pronounced than that in the expected excess return. Intuitively, because uncertainty about the stock's payoff declines, investors' demand goes up, so that—for markets to clear—the Sharpe ratio (as the equilibrium incentive to buy the stock) declines. Finally, note that, in the case of an uninformative bond market, the expected excess return, return volatility and the Sharpe ratio are all unrelated to the mean supply in the bond market and, hence, the rate of interest.

Overall, these results highlight that any changes in the characteristics of the bond market have important implications for price informativeness and risk borne by investors. In particular, variations in the mean and the "noise" in the bond market not only affect the rate of interest but also influence the stock market—through the informativeness of the price.

4 The Impact of Monetary and Fiscal Policies on Informational Efficiency

In this section, we incorporate a government into the economy. This addition allows us to relate interest rates to monetary and fiscal policies, and thus to speak to the influence of these policies on informational efficiency. We first briefly describe the economic framework in the presence of a government, then characterize the equilibrium and, finally, discuss the impact of government policies on equilibrium outcomes.

4.1 Economic Framework

The setting is an extension of the model described in Section 1. The main difference is that we allow for government spending and taxation as well as money. As a result, we now distinguish between real variables and nominal variables.

The Government

Our model of the government is purposely simple: it is a neoclassical model, in which money is neutral (so real variables are determined independently of nominal variables), Ricardian equivalence holds (so agents internalize the government's budget constraints when making decisions), and government policies are exogenous and credible. These policies satisfy the government's budget constraints, which, in our 2-period economy, implies that bonds and money issued in period 1 are redeemed in full in period 2.[14]

More specifically, the government consumes goods in periods $t = 1$ and $t = 2$, and finances its spending with a mix of taxes, debt and money. It collects T_{it} (goods) from (rational) agent i through lump-sum taxes, and T_t in aggregate. It issues real risk-free bonds in period1, at a (real) price of P_y goods per bond; the bond pays out one unit of the good in period 2. Default on government debt does not occur because, under CARA utility, there is no limit to how much taxes can be collected from agents (since their consumption can be negative). These bonds can be interpreted as those we analysed in the previous sections of the paper. The government also prints money in period 1, which it redeems in period 2. We assume that, in period 1, the government commits credibly to target levels for inflation (i.e., the period-2 good's price) and for tax proceeds (T_{2i} for every agent i).

Investors

As before, the population consists of rational and noise traders. Whereas rational traders are represented as optimizing agents, the behaviour of rational traders is not explicitly modelled and characterized instead by their residual (random) demands for assets.

Because money is dominated as a store of value (to the extent that bonds pay positive nominal interest), we introduce a benefit of holding money by assuming that agents derive utility from the quantity of real money balances they hold, which equals the number of goods their stock of money could purchase in period 1.[15] Specifically, rational investors

[14]A straightforward extension of the model is to assume these policies are chosen by the government to maximise a social welfare function.

[15]This is a commonly used shortcut to model the usefulness of money as a medium of exchange. It captures the notion that the higher the purchasing power of an agent's money holdings, the lower is the disutility cost associated with exchange, which results in higher overall utility.

have preferences of the following type:

$$U_i(C_{i,1}, C_{i,2}) = -\frac{1}{\gamma}\exp\big(-\gamma\, C_{i,1}\big) + v\left(\frac{M_i}{P_1}\right) + \beta\, E\left[-\frac{1}{\gamma}\exp\big(-\gamma\, C_{i,2}\big)\,\big|\,\mathcal{F}_i\right], \tag{16}$$

where P_t denotes the price of the good in period t ($t \in \{1,2\}$), M_i denotes money holding, and v is an increasing and concave function of agent i's real money balance in period 1, M_i/P_1.

In addition to the stock and bond market noises, we assume that the supply of money is noisy. Accordingly, we denote θ_M the residual random supply of money, i.e., the supply of money minus noise traders' demand. $\theta_M \sim \mathcal{N}(E_{\theta_M}, 1/\tau_{\theta_M})$, where E_{θ_M} and τ_{θ_M} denote the prior mean and precision of money supply. Moreover, it is uncorrelated with the other supply shocks, θ_X, θ_Y, and with the stock's payoff, Π. With three price signals, three sources of noise are needed to prevent prices from being perfectly revealing.

Equilibrium Definition

The objective of each rational investor i is to maximize expected utility (16) subject to the following budget equations:

$$C_{i,1} + X_i\, P_X + Y_i\, P_Y + \frac{M_i}{P_1} = W_{i,1} - T_{i,1}, \quad \text{and} \quad C_{i,2} = X_i\,\Pi + Y_i + \frac{M_i}{P_2} - T_{i,2}. \tag{17}$$

The budget equations are expressed in *real terms* and differ from those in (2) only in that rational investors now also hold money (M_i) and must pay taxes ($T_{i,1}$ and $T_{i,2}$).

The equilibrium is defined as in Section 1, with the exception that rational investors, in addition to consumption $\{C_{i,1}, C_{i,2}\}$ and portfolio holdings $\{X_i, Y_i\}$, now also choose their money holdings $\{M_i\}$. Correspondingly, in addition to the bond and stock markets (3), the money market clears, that is, $\int M_i di = \theta_M$.[16] Also, it is important to point out that, in

[16] By Walras' law, clearing in the bond, the stock and the money markets guarantees market clearing in the goods markets. Aggregating a rational investors budget constraints yields $\int C_{i,1}di + \left(T_1 + \frac{\theta_Y}{R_f} + \frac{\theta_M}{P_1}\right) = \Pi_1\,\theta_X$ in period 1, $\int C_{i,2}di + \left(T_2 + \theta_Y + \frac{\theta_M}{P_2}\right) = \Pi\,\theta_X$ in period 2. Intuitively, the terms in brackets on the

equilibrium, the price of the good now also serves as a price signal (in addition to the bond and stock prices). This is because the good no longer serves as a numeraire (money does). As a result, $\mathcal{F}_i = \{S_i, P_X, R_f, P_1\}$.

4.2 Optimal Consumption, Portfolio and Money Choices

Maximizing agent i's expected utility (16) subject to the budget constraints in (17), yields the following optimal consumption, portfolio and money choices.

Theorem 5. *Conditional on an investor's posterior beliefs, the endowed wealth $W_{1,i}$, the stock price P_X, the risk-free rate R_f and the good's price P_1, the optimal consumption and portfolio choices in period $t = 1$ are given by (11), (12), and (13). In addition, the investor's optimal money demand is characterized by:*

$$v'\left(\frac{M_i}{P_1}\right) = \left(R_f - \frac{P_1}{P_2}\right) \exp\bigl(-\gamma\, C_{i,1}\bigr). \tag{18}$$

The optimal consumption and portfolio choices coincide with those derived in our main framework—absent a government. The novel condition is given by (18) which characterizes the investor's money holding, M_i. Note that the term $\left(R_f - \frac{P_1}{P_2}\right)$ in this equation represents the (real) return on the bond in excess of the (real) return on money. Hence, at the optimum, the marginal benefit of money is equated to the marginal cost of forgoing an investment in the bond. Because the agent derives utility from holding money, a non-zero money balance is compatible with the bond yielding a higher return than money.[17]

left-hand side of both equations represent the consumption of the government which, together with rational investors' consumption, equals aggregate supply of the good, given on the right-hand side of the equations.

[17]Without money in the utility function ($v = 0$), a non-zero demand for money (interior solution) requires that $\frac{P_1}{P_2} = R_F$, i.e., money and the bond are perfect substitutes, so that the agent is indifferent between the two assets. Else, money is dominated by the bond so the agent chooses to hold no money (corner solution).

4.3 Equilibrium

Aggregating the rational investors' demand for the stock, the bond and money and imposing market-clearing, yields the following market-clearing conditions:

$$P_X\, R_f = \int E[\Pi \,|\, \mathcal{F}_i]\, di + \frac{\gamma\, \theta_X}{\tau}, \tag{19}$$

$$\frac{1}{\gamma} \log(\beta\, R_f) = \frac{1}{2} \int \frac{\left(E\,[\Pi \,|\, \mathcal{F}_i] - R_f\, P_X\right)^2}{\gamma\, Var\, (\Pi \,|\, \mathcal{F}_i)} di + (R_f\, P_X - \Pi_1)\; \theta_X$$
$$+ \left[\frac{1 + R_f}{R_f}\, \theta_Y + T_1 - T_2 + \frac{P_1 + P_2}{P_2}\, \frac{\theta_M}{P_1}\right], \tag{20}$$

$$\int \log\left(v'\left(\frac{M_i}{P_1}\right)\right)\, di = \log\left(1 - \frac{P_1}{R_f\, P_2}\right) - \gamma\, \theta_X \Pi_1 + \gamma \left[T_1 + \frac{\theta_Y}{R_f} + \frac{\theta_M}{P_1}\right]. \tag{21}$$

The market-clearing condition for the stock market (19) has the traditional form and coincide with that in the models without the government. The market-clearing condition for the bond market (20) is also closely related to that in the absence of the government (see (15)) but contains additional terms related to the taxes ($T_1 - T_2$) as well as real money supply (θ_M/P_1).

The last condition (21) is novel. It relates the aggregate real money balance of rational traders in period 1 to the nominal interest rate, the residual supplies of the stock and bond, and taxes. Its implications conform to what one expects from a neoclassical monetary model. First, the price level is proportional to the residual money supply.[18] Second, the real money balance decreases in the nominal interest rate, $\frac{R_f\, P_2}{P_1}$, as the opportunity cost of money rises. Finally, it increases in rational investors' initial wealth, $\theta_X \Pi_1$, and decreases in their tax payment, T_1, and their bond savings, θ_Y/R_f, because agents wish to hold more money when they consume more. Most importantly given our focus, equation (21) implies that the good's price in period 1, P_1, is a function of the stock supply, θ_X.

Together with the expressions for the rational investors' posterior beliefs, these three market-clearing conditions characterize the equilibrium in the economy. As in the preceding

[18] For example, if Θ_M and P_2 double, that is, the value of the real money balance in period 2 stays constant, then P_1 doubles.

section, it is not possible to identify the equilibrium in closed-form because the rational investors' inference problem involves non-linear functions of the state variables (Π, θ_X, θ_Y and θ_M). Hence, we again rely on a numerical algorithm to obtain the equilibrium.

As before, the market-clearing condition for the bond (20) "connects" the realizations of the residual supplies of the bond, θ_Y, the stock, θ_X, and money supply, θ_M. The presence of the residual money supply in this condition, θ_M, renders the inference problem more complicated. On the other hand, investors also observe another signal, namely, the period-1 good's price, P_1, which also conveys information. To see this, observe that the market-clearing condition for money (21) also connects the residual supplies in the three markets. Hence, together, the bond and money market signals allow investors to form more precise posterior beliefs about the supply in the stock market which, in turn, allows them to extract more payoff information from the stock's price.

We point out that Ricardian Equivalence holds in our economy, the noise that ultimately blurs prices is government consumption. To see this, note that terms in squared brackets in (20) and (21) can be written as functions of government consumption in periods 1 and 2. In particular, define residual government consumption, that is, the consumption of the government that is funded by rational traders through their taxes, their purchase of bonds and their holding of money as $\theta_{G_1} \equiv T_1 + \theta_Y/R_f + \theta_M/P_1$ in period 1 and $\theta_{G_2} \equiv T_2 - \theta_Y + \theta_M/P_2$ in period 2. Then, the terms in squared brackets in (20) and (21) can be simply written as $\theta_{G_1} - \theta_{G_2}$ and θ_{G_1}, respectively. Thus, all that matters for the real interest rate, R_f, and real money balances, θ_M/P_t, is (residual) government spending, θ_{G_1} and θ_{G_2}. Conditional on these values, the timing and amounts of taxes, T_t, are irrelevant, as are the (residual) supplies of bond and money from the government, θ_Y and θ_M. Conversely, noise in the residual government consumption, whatever its source, suffices to prevent the full revelation of information.

Variable	Obs.	Mean	SD	Median	Min	Max
Nominal Rate	48	0.0687	0.0266	0.0639	0.0282	0.1386
Real Rate (realized	48	0.0275	0.0282	0.027	-0.045	0.1033
Real Rate (AR1)	48	0.0324	0.0178	0.0303	-0.0035	0.0806
Price Informativen	48	0.0432	0.0109	0.0438	0.0177	0.0641
Volatility	48	0.0091	0.0043	0.008	0.0035	0.0277

Table 1: Summary statistics.

5 Empirical Analysis

In this section, we evaluate empirically the central prediction of our model concerning the effect of interest rates on the information about the stock market. For that purpose, we employ the stock price informativeness measure developed by Bai, Philippon, and Savov (2016), which matches our model closely. A time-series of price informativeness is obtained by estimating, every year, a cross-sectional regression of future earnings on current stock prices. A higher coefficient estimate indicates that current stock prices track future stock earnings more closely, i.e., that they contain more fundamental information. Our theory predicts that this coefficient estimate increases in the level of interest rates.

5.1 Data and Estimation Procedure

We follow the procedure for computing Price Informativeness outlined in Bai, Philippon, and Savov (2016). At the end of March every year, we estimate a cross-sectional regression of the ratio of earnings (EBIT) five-years ahead to current assets (a proxy future earnings) on the log of the ratio of current market capitalization to current assets (a proxy for current stock prices), controlling for current earnings and industry (one-digit SIC code indicator). Our sample of stocks consists of S&P 500 nonfinancial firms, which represent the bulk of the value of the US corporate sector, over the period 1962-2009 (as in Bai, Philippon, and Savov (2016)). Thus, we construct an annual times-series of Price Informativeness (covering the period 1962 to 2009).

Treasury bond data are obtained from the Federal Reserve Board's H.15 Report. We focus our analysis on 10-year bonds because they have the longest available maturity. We

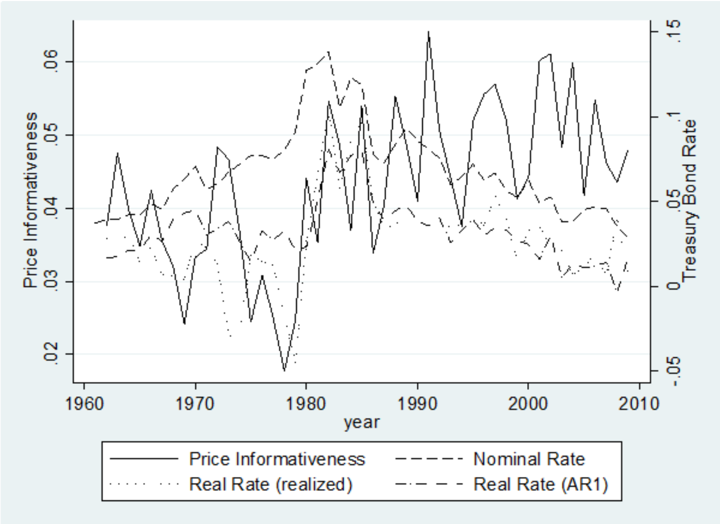

Figure 8: Time series of Treasury bond yields and price informativeness. 10-year Treasury bond yields and price informativeness are plotted over our sample period. All yields are for 10-year Treasury bonds. Real yields are obtained by deducting from nominal yields inflation expectation estimated based on realised inflation (dotted line), and a rolling window AR(1) model (dashed line). Price informativeness (solid line) is estimated as in Bai, Philippon, and Savov (2016) using S&P500 firms and a forecast horizon of 5 years. Stock price informativeness and interest rates are measured contemporaneously, as of the end of March of each year, using annual data from 1962 to 2009.

conduct our tests on both nominal and real rates. Real rates are computed by deducting expected inflation from nominal rates. Expected inflation is estimated in two ways, based on i) realised inflation, and ii) an AR(1) model. Stock price informativeness and interest rates are measured contemporaneously, as of the end of March of each year.

Because our model holds constant the volatility of fundamentals, we shall include as a control variable in some regressions the realised volatility of the S&P 500 index, estimated as the standard deviation of daily S&P 500 returns measured over a rolling 12-month window, so from April of the previous year to March of the current year. The times series of price informativeness and interest rates are displayed in Figure 8, and summary statistics presented in Table 1.

Because the data are serially correlated, we regress annual changes in Price Informativeness on annual changes in interest rates. This also allows to neutralise the time trend in Price Informativeness that Bai, Philippon, and Savov (2016) document. The regression takes the form:

$$\Delta PI_t = \alpha + \beta\,\Delta R_{f,t} + \gamma\,\Delta VOL_t + \epsilon_t \tag{22}$$

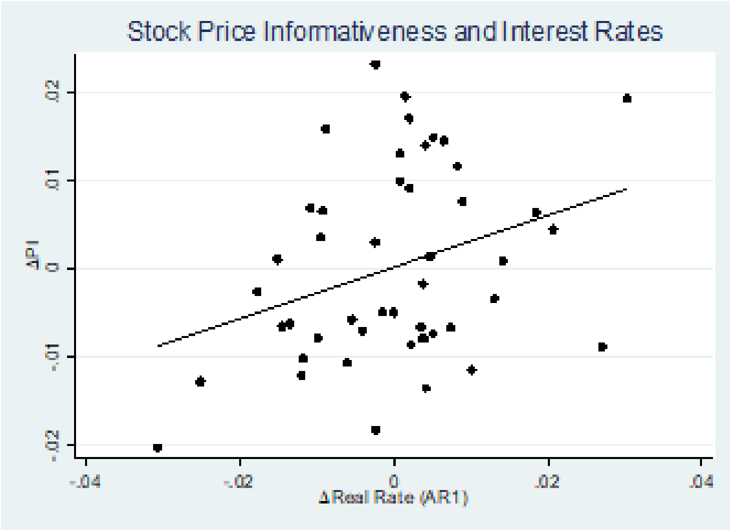

Figure 9: Annual changes in price informativeness and Treasury bond yields. Annual changes in price informativeness are plotted against annual changes in the 10-year Treasury bond yields. Yields are those of 10-year Treasury bonds. Real yields are obtained by deducting from nominal yields inflation expectation estimated based a rolling window AR(1) model. Price informativeness is estimated as in Bai, Philippon, and Savov (2016) using S&P500 firms and a forecast horizon of 5 years. Stock price informativeness and interest rates are measured contemporaneously, as of the end of March of each year, using annual data from 1962 to 2009.

where PI, R_f and VOL denote price informativeness, the interest rate and volatility, respectively.

5.2 Results

The results, displayed in Figure 9 and in Table 2, conform to the model predictions: Price Informativeness rises with interest rates. In terms of economic magnitude, our findings indicate a sizable effect: based the OLS coefficient estimate for nominal rates without controlling for volatility (0.27 in Column 1 of Table 2), a one standard-deviation increase in interest rates (2.7%) leads to an increase in PI of about two thirds (0.66=(2.7%)(0.27)/(0.011)) of a standard-deviation.

Our findings hold up to a series of robustness checks. They obtain for both nominal and real interest rates (Table 2), for shorter-maturity bonds (1,3 and 5 years), and for shorter forecast horizons (i.e., estimating Price Informativeness from a cross-sectional regression of earnings 3 years ahead rather than 5). They are robust to the inclusion of volatility of the S&P 500 index as a control. In fact, controlling for volatility strengthens the effect.

	Nominal Rate		Real Rate (realized)		Real Rate (AR1)	
	(1)	(2)	(3)	(4)	(5)	(6)
ΔR	0.264**	0.293**	0.129*	0.135*	0.292**	0.316**
	[2.08]	[2.29]	[1.93]	[2.01]	[2.29]	[2.46]
ΔVolatility		0.464		0.381		0.447
		[1.28]		[1.05]		[1.25]
Constant	0.0003	0.0001	0.0003	0.0001	0.0002	0.0000
	[0.19]	[0.04]	[0.18]	[0.07]	[0.14]	[0.00]
Obs.	47	47	47	47	47	47
R^2	9%	12%	8%	10%	10%	14%

Table 2: Regressions of price informativeness on Treasury bond yields. This table displays results from OLS regressions of annual changes in price informativeness, denoted ΔPI, on annual changes in Treasury bond yields, denoted ΔR, controlling for annual changes in volatility, denoted ΔVOL. Yields are those of 10-year Treasury bonds. Real yields are obtained by deducting from nominal yields inflation expectation estimated based on realised inflation, and a rolling window AR(1) model. Price informativeness is estimated as in Bai, Philippon, and Savov (2016) using S&P500 firms and a forecast horizon of 5 years. Stock price informativeness and interest rates are measured contemporaneously, as of the end of March of each year, using annual data from 1962 to 2009. Volatility is the realised volatility of the S&P 500 index and is estimated as the standard deviation of daily S&P 500 returns measured over the past 12 months ending at the end of March of the year. Columns 1 & 2 display estimates for nominal yields. Columns 3 to 6 display estimates for real yields, based on realised inflation in Columns 3 & 4, and on a rolling window AR(1) model in Columns 5 & 6. t-statistic are displayed in square brackets. Statistical significance at the 1%, 5% and 10% level is indicated by ***, **, *, respectively.

They are also robust to the estimation procedure. We find that the residuals of our regressions, despite differencing, continue to display some serial correlation. Therefore, we adjust standard errors using the Prais-Winsten procedure which corrects for first-order serial correlation, as well as the Newey-West correction with lags ranging from 1 to 10 years. We also estimate a median regression to reduce the influence of extreme observations and to guard against the non-normality of residuals (e.g., stock market crashes). In all instances, we obtain similar results. These are reported in Table A1 in the Appendix.

6 Conclusion

In this paper, we illustrate how rational investors use information contained in interest rates to learn about stock-market fundamentals. For that purpose, we develop a competitive noisy rational expectations equilibrium model in which the rate of interest is determined by supply and demand and, consequently, conveys information.

We demonstrate that rational investors use the information in interest rates to form posterior beliefs about the stock's supply. As a result of their more precise beliefs about its supply, rational investors can then more precisely infer the stock's payout from its price. Hence, in the presence of an endogenous rate of interest, price informativeness increases. Importantly, however, the strength of this effect is positively related to the interest rate. In particular, a higher interest rate (lower bond price) reduces the importance of the (dollar) bond-market supply in the bond-market clearing condition (which guarantees that, on aggregate, investors' demand and supply of goods is balanced) which makes is easier for rational investors to learn about the stock's supply. Our empirical analysis finds robust evidence in favor of this prediction.

We then analyze how variations in the characteristics of the bond market, in particular, the mean supply and its precision, affect informational efficiency in the stock market and, hence, asset prices. Naturally, the interest rate is increasing in the mean supply of the bond which in turn, leads to an increase in price informativeness (as discussed above). This increase partially offsets the decline in the stock price due to the higher interest rate. Moreover, it leads to a decline in the stock's return volatility, expected excess return and Sharpe ratio.

We also study an extension of our main economic framework incorporating government spending, taxation and money. In this model, the rate of interest—together with the endogenous price of the consumption good—again allows rational investors to learn about the stock's supply. Importantly, this setting allows us to study how fiscal and monetary policies affect informational efficiency in the stock market through their impact on the bond market.

	Prais-Winsten			Newey West (2 lags)			Newey West (5 lags)			Median		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
ΔR	0.229^{**}	0.116^{*}	0.270^{**}	0.293^{***}	0.135^{**}	0.316^{***}	0.293^{***}	0.135^{**}	0.316^{***}	0.434^{**}	0.231^{**}	0.341^{*}
	[2.06]	[1.97]	[2.26]	[3.14]	[2.14]	[3.07]	[4.12]	[2.07]	[3.38]	[2.32]	[2.23]	[1.81]
ΔVolatility	0.451	0.387	0.449	0.464	0.381	0.447	0.464	0.381	0.447	0.793	0.791	0.433
	[1.37]	[1.19]	[1.38]	[1.37]	[0.97]	[1.34]	[1.21]	[0.85]	[1.20]	[1.49]	[1.41]	[0.83]
Constant	0.0000	0.0000	0.0000	0.0001	0.0001	0.0000	0.0001	0.0001	0.0000	-0.0017	0.0000	-0.0028
	[0.02]	[0.03]	[0.01]	[0.07]	[0.10]	[0.00]	[0.09]	[0.16]	[0.01]	[-0.76]	[-0.01]	[-1.23]
Obs.	47	47	47	47	47	47	47	47	47	47	47	47

Table A1: Robustness checks. This table displays robustness tests for our baseline regressions of annual changes in price informativeness, denoted ΔPI, on annual changes in Treasury bond yields, denoted ΔR, controlling for annual changes in volatility, denoted ΔVOL. Yields are those of 10-year Treasury bonds. Real yields are obtained by deducting from nominal yields inflation expectation estimated based on a rolling window AR(1) model. Price informativeness is estimated as in Bai, Philippon, and Savov (2016) using S&P500 firms and a forecast horizon of 5 years. Stock price informativeness and interest rates are measured contemporaneously, as of the end of March of each year, using annual data from 1962 to 2009. Volatility is the realised volatility of the S&P 500 index and is estimated as the standard deviation of daily S&P 500 returns measured over the past 12 months ending at the end of March of the year. Columns 1 to 6 display estimates that correct for serial correlation in residuals, using the Prais-Winsten procedure (Columns 1 & 2) and the Newest-West correction with lags of 2 and 5 years (Columns 3 to 6). Columns 7 & 8 display estimates form a median regression. t-statistic are displayed in square brackets. Statistical significance at the 1%, 5% and 10% level is indicated by ***, **, *, respectively.

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